The Shape of the UNNS Substrate: Admissibility Geometry, Stratified Manifolds, and Observability through Invariance

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Abstract

We address the question: *does the UNNS Substrate have a shape, and can that shape be observed?* The answer is affirmative on both counts, but indirect: the shape is not visible as a geometric object in ordinary space; it is inferred through *invariance geometry*, the pattern of descent stability under admissible operator families. We formalize the admissibility region as a stratified manifold in the perturbation envelope plane (σ_P, δ_P) and, more generally, as an open submanifold of the admissible operator space $\mathcal{B}(X)$ equipped with the operator norm topology. We prove that the rigid admissibility region is convex in σ_P, that the degeneracy phase surface $\nu(\sigma_P)$ is monotone but not convex, and that certified regions for any fixed inversion budget are rectangles (hence convex). The boundary geometry is governed by a matching number $\nu(V)$ on vulnerable adjacent gaps, giving a discrete stratification of admissibility space. Empirical instantiation on the LXV seismic chamber suite (Kumamoto, Ridgecrest, El Mayor) confirms the predicted phase structure across two orders of magnitude in the Rigidity Modulus R, demonstrating that near-fault station geometry — not event magnitude — determines the admissibility margin and hence the substrate phase.

Keywords: UNNS Substrate, admissibility geometry, stratified manifold, invariance geometry, rigidity modulus, phase boundary, operator manifold, seismic rank stability.

Contents

1 Introduction

The UNNS Substrate framework asserts that every observable co-seismic displacement hierarchy is a realization inside an *admissible operator family*, and that structural stability is the statement that the induced signature descends through the quotient by admissible transforms. A central open question, both for theoretical clarity and for empirical tractability, is: *what does the UNNS Substrate look like*?

Answering this question requires precision. "Shape" in the substrate context does not mean a geometric object in physical space. The substrate is not a fault surface, a focal mechanism, or a station network layout. Rather, the substrate is an abstract space — the space of admissible operator families — and its shape is the geometry of the *admissibility region* inside that space.

This paper provides three interlocking contributions toward a complete answer.

Contribution 1: Admissibility Region as a Manifold. We show that the rigid admissibility region $\mathcal{A}_{\text{rig}}$ in the perturbation envelope plane (σ_P, δ_P) is an open submanifold of $\mathbb{R}^2$, hence a smooth 2-dimensional manifold. The full admissibility landscape is richer: it is a *stratified manifold*, with strata separated by codimension-one critical lines at $\sigma_P = \Delta_k/2$ for each adjacent gap Δ_k.

Contribution 2: Convexity Analysis. We prove that the rigid region is convex in σ_P (it is an interval). We prove that the degeneracy phase surface $\nu(\sigma_P)$ is monotone but *not* convex, being a step function. We prove that for any fixed inversion budget k, the certification region $\mathcal{C}_k$ is a rectangle, hence convex.

Contribution 3: Operator Manifold Generalization. We generalize from the envelope plane to the full admissible operator manifold $\mathcal{M} \subset \mathcal{B}(X)$, showing that rigidity corresponds to an open region $\mathcal{M}_{\text{rig}}(x)$, boundary degeneracy corresponds to crossing stratification hypersurfaces $\mathcal{H}_k$, and convexity is replaced by monotonicity along increasing-envelope paths.

Throughout, empirical validation is provided by the LXV seismic chamber suite, which instantiates all three predicted substrate phases — rigid, boundary degenerate, and (theoretically) fragmented — with three earthquake events spanning two orders of magnitude in the Rigidity Modulus R.

2 Preliminaries: Structural Signature and Perturbation Envelope

Let $\mathcal{N}$ be a finite station set, $|\mathcal{N}| = N$. Let $S_w(s)$ denote the co-seismic horizontal displacement magnitude at station s under smoothing window w, and let $\theta_w(s) := \arg(\mathbf{u}_w(s))$ denote the displacement azimuth under the same window. Let $w_0 = 1$ denote the baseline window throughout.

Definition 2.1 (Inversion Distance)**.** For window w, define

$$D(w) = \#\{(i,j) : R_{w_0}(i) < R_{w_0}(j) \text{ but } R_w(i) > R_w(j)\},$$

where R_w is the total order induced by S_w.

Definition 2.2 (Adjacent Gap)**.** For baseline ranking R_{w_0} with ordered magnitudes $S_1(i_1) \geq S_1(i_2) \geq \cdots \geq S_1(i_N)$, define the adjacent gaps

$$\Delta_k = S_1(i_k) - S_1(i_{k+1}), \quad k = 1, \ldots, N-1.$$

Definition 2.3 (Perturbation Envelope)**.** The perturbation family $\mathcal{W}$ is *perturbation-admissible* with scalar envelope $\sigma_P \geq 0$ and angular envelope $\delta_P \geq 0$ if, for every $s \in \mathcal{N}$ and every $w \in \mathcal{W}$:

$$|S_w(s) - S_1(s)| \leq \sigma_P \quad \text{and} \quad |\theta_w(s) - \theta_1(s)| \leq \delta_P.$$

Definition 2.4 (Rigidity Moduli)**.** The *magnitude rigidity modulus* and *geometric rigidity modulus* are, respectively:

$$R_{\text{mag}} := \frac{\min_k \Delta_k}{2\sigma_P}, \qquad R_{\text{geo}} := \frac{\Theta_{\min}}{2\delta_P},$$

where $\Theta_{\min} = \min_{i \neq j} \inf_{s \in C_i, t \in C_j} |\theta_1(s) - \theta_1(t)|$ is the baseline inter-cluster angular separation. The *admissibility margin* is $\mathcal{A} := \min(R_{\text{mag}}, R_{\text{geo}})$.

These two moduli constitute the phase coordinate vector $\mathbf{R} = (R_{\text{mag}}, R_{\text{geo}})$, and the "shape" of the substrate is the geometry of the admissibility region in $\mathbf{R}$-space, as formalized in the sections below.

3 The Three Geometric Components of Shape

The substrate shape has three interlocking geometric components.

Order Geometry. The baseline adjacent gaps $\{\Delta_k\}_{k=1}^{N-1}$ define a one-dimensional "gap spectrum" that encodes the separability of the station network in displacement space. A dense gap spectrum (small $\Delta_{\min}$) implies fragility; a sparse spectrum (large $\Delta_{\min}$) implies robustness.

Perturbation Envelope. The pair (σ_P, δ_P) defines a two-dimensional envelope parameter space $\mathcal{P} = \mathbb{R}^2_{\geq 0}$. The choice of σ_P and δ_P reflects both the measurement noise level and the extent of the admissible operator family.

Matching Geometry. As σ_P increases, a growing set of gaps becomes *vulnerable*:

$$V(\sigma_P) := \{k : \Delta_k \leq 2\sigma_P\}.$$

The matching number $\nu(V(\sigma_P))$ — the maximum size of a disjoint subset of $V(\sigma_P)$ — governs the tight inversion budget. The adjacency structure of $V(\sigma_P)$ within the gap spectrum is the *degeneracy geometry*.

Together these three components determine a geometric object: the admissibility region in $\mathcal{P}$, stratified by ν-level sets.

4 Admissibility Region as a Stratified Manifold

4.1 Rigid Admissibility Region

Fix an event E and its baseline station configuration. Define the *envelope parameter space* $\mathcal{P} := \{(\sigma_P, \delta_P) \in \mathbb{R}^2_{\geq 0}\}$. Define the margin functions

$$g(\sigma_P) := \Delta_{\min} - 2\sigma_P, \qquad h(\delta_P) := \Theta_{\min} - 2\delta_P.$$

Definition 4.1 (Rigid Admissibility Region)**.**

$$\mathcal{A}_{\text{rig}} := \{(\sigma_P, \delta_P) \in \mathcal{P} : g(\sigma_P) > 0 \ \wedge \ h(\delta_P) > 0\}.$$

Lemma 4.2 (Manifold Structure of $\mathcal{A}_{\text{rig}}$)**.** *$\mathcal{A}_{\text{rig}}$ is an open subset of $\mathbb{R}^2$ and hence a smooth 2-dimensional submanifold of $\mathcal{P}$. Its boundary in $\mathcal{P}$ is contained in the union of the two lines*

$$\partial\mathcal{A}_{\text{rig}} \subseteq \{\sigma_P = \Delta_{\min}/2\} \cup \{\delta_P = \Theta_{\min}/2\}.$$

Proof. g and h are continuous affine functions; strict inequalities define an open set in $\mathbb{R}^2$. □

4.2 Degeneracy Stratification

Definition 4.3 (Vulnerable Set and Matching Number)**.** For $\sigma_P \geq 0$, define

$$V(\sigma_P) := \{k \in \{1, \ldots, N-1\} : \Delta_k \leq 2\sigma_P\}, \qquad \nu(\sigma_P) := \nu(V(\sigma_P)),$$

where $\nu(V)$ is the maximum size of a disjoint subset of V (the matching number on the induced path graph, computed in $O(|V|)$ time by a greedy left-to-right scan).

Definition 4.4 (Degeneracy Strata)**.** For each integer $m \in \{0, 1, \ldots, \lfloor (N-1)/2 \rfloor\}$:

$$\mathcal{S}_m := \{(\sigma_P, \delta_P) \in \mathcal{P} : \nu(\sigma_P) = m \ \wedge \ h(\delta_P) \geq 0\}.$$

Theorem 4.5 (Stratified-Manifold Picture)**.** *Let $\mathcal{A}_{\text{deg}} := \bigcup_m \mathcal{S}_m$ be the degeneracy-admissible region. Then $\mathcal{A}_{\text{deg}}$ is a* stratified *subset of $\mathcal{P}$:*

(i) Each stratum $\mathcal{S}_m$ is a finite union of semi-open rectangles in $\mathbb{R}^2$.

(ii) Stratum boundaries occur only at the critical values $\sigma_P = \Delta_k/2$ ($k = 1, \ldots, N-1$) and $\delta_P = \Theta_{\min}/2$.

Proof. As σ_P varies, $V(\sigma_P)$ changes only when $2\sigma_P$ crosses some Δ_k. Between such crossings $V(\sigma_P)$ is constant, hence $\nu(\sigma_P)$ is constant. Each level set $\{\nu(\sigma_P) = m\}$ is therefore a union of intervals in σ_P; intersecting with $h(\delta_P) \geq 0$ yields unions of rectangles. □

The stratified structure gives the substrate shape a "staircase" profile in $\mathcal{P}$: a rigid plateau, a narrow boundary ridge, and (theoretically) a chaotic basin at large σ_P.

5 Convexity Analysis

5.1 Convexity of the Rigid Region

Theorem 5.1 (Convexity of $\mathcal{A}_{\text{rig}}$ in σ_P). *Fix δ_P with $h(\delta_P) > 0$. The cross-section*

$$\{\sigma_P \geq 0 : (\sigma_P, \delta_P) \in \mathcal{A}_{\text{rig}}\} = \left[0, \frac{\Delta_{\min}}{2}\right)$$

is convex.

Proof. $g(\sigma_P) > 0$ is equivalent to $\sigma_P < \Delta_{\min}/2$. □

5.2 Phase Surface: Monotonicity but Not Convexity

Theorem 5.2 (Monotonicity and Non-Convexity of $\nu(\sigma_P)$). *The function $\nu(\sigma_P)$ is monotone nondecreasing in σ_P. Moreover, $\nu(\sigma_P)$ is a right-continuous step function and therefore is not convex (nor concave) except in degenerate cases.*

Proof. If $\sigma_P \leq \sigma'_P$, then $V(\sigma_P) \subseteq V(\sigma'_P)$. Any disjoint subset of $V(\sigma_P)$ is also a disjoint subset of $V(\sigma'_P)$, giving $\nu(\sigma_P) \leq \nu(\sigma'_P)$. Since $V(\sigma_P)$ changes only when $2\sigma_P$ crosses some Δ_k, ν is piecewise constant. A nonconstant step function cannot satisfy the Jensen inequality. □

5.3 Convexity of Certified Regions

Corollary 5.3 (Certification Region is Convex). *Fix a budget $k \geq 0$ and define*

$$\mathcal{C}_k := \{(\sigma_P, \delta_P) \in \mathcal{P} : \nu(\sigma_P) \leq k \ \wedge \ h(\delta_P) \geq 0\}.$$

By monotonicity of ν, the set $\{\sigma_P : \nu(\sigma_P) \leq k\}$ is an interval $[0, \sigma_k)$ for some threshold $\sigma_k \geq 0$. Therefore

$$\mathcal{C}_k = [0, \sigma_k) \times [0, \Theta_{\min}/2],$$

which is a rectangle, hence convex.

Summary of Convexity Results. The rigid admissibility region is an interval in σ_P (convex). The phase surface $\nu(\sigma_P)$ is monotone but non-convex (step function). For any fixed inversion budget k, the certification region $\mathcal{C}_k$ is a rectangle (convex). This trichotomy — rigid convexity, non-convex phase surface, convex certified budgets — is the precise convexity structure of the substrate shape.

6 Generalization: The Admissible Operator Manifold

The envelope plane (σ_P, δ_P) is a derived coordinate system on the orbit of a single event. We now show that the substrate shape is more fundamentally described as a submanifold of the full admissible operator space.

6.1 Setup

Let X be the event representation space (detrended displacement time series restricted to the LXV protocol), modeled as a real Banach space with norm $\|\cdot\|_X$. Let $\mathcal{B}(X)$ denote the Banach algebra of bounded linear operators on X with operator norm $\|T\|_{\mathrm{op}} := \sup_{\|x\|_X=1} \|Tx\|_X$.

Definition 6.1 (Admissible Operator Manifold)**.** An *admissible operator family* is a C^1 injective map $\Phi : \Omega \to \mathcal{B}(X)$, where $\Omega \subset \mathbb{R}^d$ is open, such that:

(i) For each $\omega \in \Omega$, $T_\omega := \Phi(\omega)$ implements only protocol-admissible transformations (centered smoothing, fixed guard convention, fixed detrending interface).

(ii) The differential $D\Phi_\omega : \mathbb{R}^d \to \mathcal{B}(X)$ is injective for all ω, so $\mathcal{M} := \Phi(\Omega) \subset \mathcal{B}(X)$ is a d-dimensional immersed submanifold of $\mathcal{B}(X)$.

We call $\mathcal{M}$ the *admissible operator manifold*.

Remark 6.2. In the LXV instance, Ω encodes window width, kernel choice, and guard convention parameters. The discrete window list $\mathcal{W}$ corresponds to a finite subset of Ω.

6.2 Envelope Projection

Fix a baseline operator $T_0 \in \mathcal{M}$ and an event representation $x \in X$. The *envelope projection* $E_x : \mathcal{M} \to \mathbb{R}^2_{\geq 0}$ is defined by

$$\sigma_x(T) := \sup_{s \in \mathcal{N}} |S(T(x); s) - S(T_0(x); s)|, \qquad \delta_x(T) := \sup_{s \in \mathcal{N}} |\theta(T(x); s) - \theta(T_0(x); s)|,$$

$$E_x(T) := (\sigma_x(T),\, \delta_x(T)).$$

The admissibility region in (σ_P, δ_P) analyzed in Sections 4–5 is the image of subsets of $\mathcal{M}$ under E_x.

6.3 Rigidity as an Open Submanifold

Let $\Delta_{\min}(x)$ and $\Theta_{\min}(x)$ denote baseline gap/separation quantities induced by $T_0(x)$.

Definition 6.3 (Rigid Region in $\mathcal{M}$)**.**

$$\mathcal{M}_{\mathrm{rig}}(x) := \{T \in \mathcal{M} : 2\sigma_x(T) < \Delta_{\min}(x) \ \wedge \ 2\delta_x(T) < \Theta_{\min}(x)\}.$$

Theorem 6.4 (Open Submanifold of Rigidity)**.** *If σ_x, δ_x are continuous on $\mathcal{M}$ in the operator-norm topology, then $\mathcal{M}_{\mathrm{rig}}(x)$ is open in $\mathcal{M}$, hence a (possibly disconnected) open d-dimensional submanifold of $\mathcal{M}$.*

Proof. σ_x and δ_x are continuous; strict inequalities define an open set. □

6.4 Boundary Stratification in Operator Space

Define, for each $T \in \mathcal{M}$:

$$V_x(T) := \{k : \Delta_k(x) \le 2\sigma_x(T)\}, \qquad \nu_x(T) := \nu(V_x(T)).$$

Theorem 6.5 (Stratification of Degeneracy in Operator Space). *Under the adjacent-swap regime and continuity of σ_x, $\nu_x : \mathcal{M} \to \mathbb{Z}_{\ge 0}$ is locally constant on the complement of the critical hypersurfaces*

$$\mathcal{H}_k := \{T \in \mathcal{M} : 2\sigma_x(T) = \Delta_k(x)\}.$$

Therefore $\mathcal{M}$ decomposes into strata with constant maximal certified inversion budget $\nu_x(T)$, forming a stratified-manifold structure with codimension-one boundaries $\bigcup_k \mathcal{H}_k$.

6.5 Monotonicity in Operator Space

Convexity is not intrinsic to $\mathcal{M}$ unless $\mathcal{M}$ is affine. The correct replacement is monotonicity along increasing-envelope paths.

Theorem 6.6 (Order Monotonicity Along Increasing-Envelope Paths). *Let $\gamma : [0,1] \to \mathcal{M}$ be a continuous path such that $\sigma_x(\gamma(t))$ is nondecreasing in t. Then $\nu_x(\gamma(t))$ is nondecreasing in t.*

Proof. If σ_x increases, V_x can only gain elements; the matching number ν is monotone under set inclusion. □

Corollary 6.7 (Certified Region is Path-Stable). *Fix $k \ge 0$ and define $\mathcal{C}_k(x) := \{T \in \mathcal{M} : \nu_x(T) \le k,\ 2\delta_x(T) < \Theta_{\min}(x)\}$. If γ is any path in $\mathcal{M}$ along which σ_x is nonincreasing, and $\gamma(0) \in \mathcal{C}_k(x)$, then $\gamma(t) \in \mathcal{C}_k(x)$ for all t.*

7 Observability of the Substrate Shape

The shape of the substrate is not directly visible. What is observed empirically is: station displacements $S_w(s)$ and $\theta_w(s)$, rank orderings R_w, inversion counts $D(w)$, cluster assignments C_w, and ARI statistics. From these observables we *compute* R_{mag}, R_{geo}, and $\nu(V)$, and from these we *reconstruct* the phase diagram.

Thus the substrate shape is *inferred through descent stability*, not seen directly.

7.1 Three Methods of Making the Shape Visible

Method A: Phase Diagram. Plot each seismic event as a point in the $(R_{\text{mag}}, R_{\text{geo}})$ plane and draw boundary lines at $R_{\text{mag}} = 1$ and $R_{\text{geo}} = 1$. The resulting diagram partitions the plane into four quadrants: fully rigid (both > 1), magnitude-degenerate, direction-degenerate, and fully fragmented.

Method B: Vulnerability Graph. Plot gap index k vs. gap size Δ_k and mark the threshold $2\sigma_P$. The pattern of adjacency among vulnerable gaps is the degeneracy geometry; the matching number ν reads off from this graph.

Method C: Operator-Orbit Visualization. For each window w, map the structural signature:

$$w \mapsto (D(w),\ \mathrm{ARI}(w)).$$

A flat orbit (constant across w) signals rigid substrate; a bifurcating orbit signals proximity to a boundary hypersurface $\mathcal{H}_k$. The shape of the orbit in (D, ARI) space is a direct visualization of substrate geometry.

7.2 What the LXV Suite Tells Us About the Shape

The LXV chamber suite provides empirical evidence for the following geometric structure of the substrate:

- **Kumamoto** sits deep in the rigid interior ($R \approx 21.6$, $D(w) = 0$ for all windows), confirming a wide rigid plateau.
- **Ridgecrest** sits in the interior but closer to the boundary ($R_{\text{eff}} \approx 4.7$), with no observed inversions.
- **El Mayor (B2)** sits on the boundary manifold ($R \approx 0.19$, one inversion observed, $\nu(V) = 1$), confirming boundary degeneracy is structurally forced.
- No fragmentation was observed within the tested admissible family.

The substrate shape inferred from these three events is: a *rigid plateau* with a *narrow boundary ridge* and an unexplored chaotic basin. This is geometric information about the substrate, not about the earthquakes.

8 The RNP Phase-Boundary Theorem and Substrate Interpretation

Theorem 8.1 (RNP Phase-Boundary Theorem)**.** *Assume the perturbation-admissible family $\mathcal{W}$ and the adjacent-swap regime* $\max_s |r_w(s) - r_1(s)| \le 1$.

(1) (Rigid descent.) *If* $\mathbf{R} > (1,1)$ *componentwise, then the structural signature* $\Sigma(E) := \{R_w,\ D(w),\ \nu(V),\ C_w\}_{w\in\mathcal{W}}$ *is invariant on each G_{adm}-orbit; equivalently, Σ descends through the quotient and is law-admissible.*

(2) (Boundary forcing.) *If* $\mathbf{R}$ *meets a phase boundary ($R_{\text{mag}} \le 1$ or $R_{\text{geo}} \le 1$), then any attempt to enforce a single rigid description fails: there exist admissible transforms under which the rigid description changes, and bounded degeneracy is structurally forced.*

(3) (Nonrigid impossibility.) *If either modulus is sufficiently small that no bounded-degeneracy certificate holds, then no orbit-invariant structural signature exists in the restricted model class and fragmentation is unavoidable.*

The RNP Phase-Boundary Theorem is an admissibility-geometry theorem:

> *Structural lawhood in the UNNS Substrate exists precisely in the interior of admissibility margin $\mathcal{A} > 1$. Boundary degeneracy corresponds to admissibility-critical configurations; the nonrigid regime corresponds to substrate phase fragmentation under admissible operator nesting.*

8.1 Structural Identification Table

Table 1 records the precise correspondence between perturbation-stability constructs and UNNS substrate objects.

Table 1: Correspondence between perturbation-stability constructs and UNNS substrate objects.

Perturbation-stability construct	**UNNS substrate object**
Smoothing window $w \in \mathcal{W}$	Admissible operator O_w
Perturbation envelope σ_P	Bounded admissible deformation radius
Rigidity modulus R	Admissibility margin in substrate phase space
Degeneracy index $\mathcal{D}$	Boundary proximity measure
Descent of Σ through Π	Quotient stability (law-admissibility)
Phase boundary $R = 1$	RNP-critical surface in the substrate
Admissibility region $\mathcal{A}_{\text{rig}}$	Open submanifold of operator space
Stratification hypersurfaces $\mathcal{H}_k$	Codimension-one phase boundaries

9 Empirical Phase Structure: LXV Suite

9.1 LXV-A: Kumamoto 2016 (Rigid Phase)

Baseline magnitudes (mm): 787.94, 453.88, 324.07, 51.44. Adjacent gaps: 334.06, 129.81, 272.63. $\Delta_{\min} = 129.81$ mm; $\sigma_P^{(\text{emp})} \approx 3.0$ mm.

$$R = \frac{129.81}{2 \times 3.0} \approx 21.6 \gg 1.$$

Observed: no inversions; Spearman = 1, Kendall = 1. Consistent with deep interior of rigid phase; Theorem 8.1(1) applies.

9.2 LXV-C2: Ridgecrest 2019 (Effective Rigid Phase)

Baseline magnitudes (mm): 679.17, 235.12, 226.22, 169.41, 44.78, 32.94. Adjacent gaps: 444.05, 8.90, 56.81, 124.63, 11.84. $\sigma_P^{(\text{emp})} \approx 6.1$ mm; global $R_{\text{global}} \approx 0.73$ (dominated by near-fault station P595). Effective modulus for the critical rank-2/rank-3 pair:

$$R_{\text{eff}} \approx \frac{56.81}{2 \times 6.1} \approx 4.7 > 1.$$

No inversion observed. This motivates pair-specific rigidity moduli (Section 10).

9.3 LXV-B2: El Mayor 2010 (Boundary Phase)

Baseline magnitudes (mm): 104.94, 49.11, 7.95, 7.46, 6.96. Adjacent gaps: 55.83, 41.16, 0.484, 0.499. $\Delta_{\min} = 0.484$ mm; $\sigma_P^{(\text{emp})} \approx 1.3$ mm.

$$R = \frac{0.484}{2 \times 1.3} \approx 0.19 \ll 1.$$

Vulnerable set $V = \{3, 4\}$ (two adjacent indices); $|V| = 2$; $\nu(V) = 1$ (only one of the two boundaries can be independently swapped). Observed: one inversion, max rank shift = 1, Kendall ≥ 0.8, Spearman ≥ 0.9. Consistent with boundary degeneracy; the chamber's budget $k_{\text{allowed}} = 1$ is the tight minimal budget $k^{\min}_{\text{allowed}} = \nu(V) = 1$.

9.4 LXV-D: El Mayor Topology (Rigid Phase)

Angular centroid separation $\Theta \approx 69^\circ$; $\delta_P < 1.5^\circ \ll \Theta/2 \approx 34.5^\circ$. Topology is rigid (ARI = 1 across all windows) as predicted. Rgeo $\gg 1$ confirms interior rigidity in the directional channel.

9.5 Summary

Table 2: Empirical phase summary for the LXV suite.

Event	$\Delta_{\min}$ (mm)	σ_P (mm)	R	$\mathcal{D}$	**Phase**
Kumamoto (A)	∼130	∼3	∼21.6	0	Rigid
Ridgecrest (C2)	57–125 (eff.)	∼6	∼4.7–10	0.2	Rigid
El Mayor (B2)	0.484	∼1.3	∼0.19	0.5	Boundary

The three events span two orders of magnitude in R despite comparable moment magnitudes ($M_w \approx 7$). This stratification encodes station network geometry and near-fault proximity distribution, not seismic energy release. The R metric is therefore a discriminating structural diagnostic beyond what magnitude alone provides.

10 Discussion

Pair-Specific Rigidity Modulus. The global σ_P is dominated by the station with the highest displacement variation. A pair-specific modulus

$$R_{(i,j)} := \frac{\Delta_{ij}}{\sigma_P^{(i,j)}}, \qquad \sigma_P^{(i,j)} = \max_w \left|[S_w(i) - S_w(j)] - [S_1(i) - S_1(j)]\right|,$$

provides tighter per-pair bounds: $\sigma_P^{(i,j)} \leq 2\sigma_P$ but is often much smaller. For Ridgecrest, the global $R_{\text{global}} = 0.73$ is misleading; the relevant rank-2/rank-3 pair has $R_{\text{eff}} \approx 4.7$, explaining the empirically observed stability.

Guard Days and Design Tradeoff. Guard days g reduce the usable epoch count m, increasing $\sigma_P(\alpha)$ and decreasing $\hat{R}(\alpha)$. This is an explicit design tradeoff: excluding post-seismic contamination comes at the cost of a smaller theoretical stability margin. An optimal guard-day choice $g^* = \arg\max_g \hat{R}(g, \alpha)$ is event-dependent.

Substrate Shape Beyond the Seismic Setting. The admissibility manifold $\mathcal{M}$ and its rigidity submanifold $\mathcal{M}_{\text{rig}}(x)$ are defined for any event representation space X and any admissible operator family. The LXV suite is one instantiation.

Other UNNS application domains — quantum algorithm diagnostics, spectroscopy, fundamental constants validation — can be similarly analyzed by identifying the appropriate analogs of σ_P, $\Delta_{\min}$, and $\nu(V)$.

11 Falsifiability

The framework admits distinct falsification classes.

Structural falsifiers (theorems are wrong).

F1. $D(w) > 0$ while $R > 1$ under the empirical envelope. This falsifies Theorem 8.1(1).

F2. Cluster reassignment (ARI < 1) while $\Theta > 2\delta_P$. This falsifies the Directional Stability Theorem.

F3. Inversion count exceeding the Degeneracy Bound $|V|(N-1)$. This falsifies the General Degeneracy Bound.

Parameter falsifiers (model fit is wrong).

F4. An observed inversion while $\hat{R}(\alpha) > 1$ for the chosen α. This falsifies the sub-Gaussian noise model or guard-day protocol.

F5. Systematic monotone dependence of $\hat{R}(\alpha)$ on w inconsistent with the $\sigma/\sqrt{w}$ decay prediction.

F6. $D(w)$ systematically exceeding $|V|$ in the boundary regime despite adjacent-only perturbations.

Degenerate non-evidence.

- Perfect ARI in a TOPO_SINGLE event provides no evidence for or against the Topology Rigidity Theorem.
- The Ridgecrest global $R_{\text{global}} = 0.73 < 1$ combined with no observed inversion is consistent with the theory (boundary regime is not a guarantee of inversion).

12 Conclusion

The UNNS Substrate has a shape, and it can be made visible, but only indirectly — through the geometry of descent stability under admissible operator families.

The shape has three components: (i) the rigid admissibility region $\mathcal{A}_{\text{rig}}$, an open submanifold of $\mathcal{P} \cong \mathbb{R}^2$, whose cross-section in σ_P is an interval (convex); (ii) the degeneracy phase surface $\nu(\sigma_P)$, a step function (monotone but not convex); (iii) the certification regions $\mathcal{C}_k$, which are rectangles (convex).

Passing to the full operator manifold $\mathcal{M} \subset \mathcal{B}(X)$, rigidity corresponds to the open submanifold $\mathcal{M}_{\text{rig}}(x)$, boundary degeneracy to crossing hypersurfaces $\mathcal{H}_k$, and convexity is replaced by monotonicity along increasing-envelope paths.

The LXV seismic chamber suite empirically confirms the predicted phase structure: Kumamoto sits deep in the rigid interior ($R \approx 21.6$), El Mayor B2 lies on the boundary manifold ($R \approx 0.19$, tight budget $\nu(V) = 1$), and Ridgecrest is effectively rigid after pair-specific modulus correction ($R_{\text{eff}} \approx 4.7$). No fragmentation was observed within the tested admissible family.

The two-order-of-magnitude spread in R across events of comparable magnitude demonstrates that the admissibility margin encodes station network geometry and near-fault configuration, not seismic energy release. The substrate shape is therefore a structural diagnostic about the observational geometry, not merely a robustness certificate for the displacement hierarchy.

Structural lawhood in the UNNS Substrate exists precisely in the interior of admissibility margin $\mathcal{A} > 1$. The boundary is not a failure mode; it is a structural feature.

A Empirical Phase Structure and Degeneracy Index (LXV Suite)

LXV-A: Kumamoto 2016

$\Delta_{\min} = 129.81$ mm; $\sigma_P \approx 3.0$ mm; $2\sigma_P = 6$ mm. All gaps $\gg 6$ mm; $|V| = 0$; $\mathcal{D} = 0$; $R \approx 21.6$. Observed: no inversions. Fully rigid phase.

LXV-C2: Ridgecrest 2019

$\sigma_P \approx 6.1$ mm; $2\sigma_P \approx 12.2$ mm. Only the 8.90 mm gap satisfies $\Delta_k \leq 12.2$ mm; $|V| = 1$; $\mathcal{D} = 1/5 = 0.2$; $R_{\text{global}} \approx 0.73$. Observed: no inversions. $\mathcal{D}$ predicts vulnerability, not necessity.

LXV-B2: El Mayor 2010

$\Delta_{\min} = 0.484$ mm; $\sigma_P \approx 1.3$ mm; $2\sigma_P \approx 2.6$ mm. Both tail gaps (0.484 and 0.499 mm) satisfy $\Delta_k \leq 2.6$ mm; $|V| = 2$ (adjacent: $k = 3$ and $k = 4$); $\nu(V) = 1$; $\mathcal{D} = 2/4 = 0.5$; $R \approx 0.19$. Observed: one inversion; max rank shift = 1. Boundary degeneracy confirmed; tight budget $k^{\min}_{\text{allowed}} = 1$.

Table 3: Degeneracy index summary (LXV suite).

Event	R	$\mathcal{D}$	**Observed**	**Regime**
Kumamoto	$\gg 1$	0	Rigid	Rigid
Ridgecrest	> 1 (eff.)	0.2	Rigid	Rigid
El Mayor	$\ll 1$	0.5	Boundary	Boundary