

Recursive Projection and Structural Admissibility: From Complex Echoes to Predictive Boundaries

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Abstract

ESC-I (Echo–Admissibility Correspondence Chamber I) is a small exploratory laboratory that develops the early UNNS complex-number programme [1, 2] into a testable mathematical framework. That programme represents complex multiplication through real recursive operators, cyclotomic structure and lattice inlaying. ESC-I (i) re-derives the real-recursion picture and classifies orbits exactly in $\mathbb{Q}$; (ii) audits the projected-stability and phase-inletting statements of the original manuscript (its §4 and §5), determines the conditions under which they hold exactly, and proves refined statements with explicit hypotheses and counterexamples; (iii) freezes and hashes admissibility rules AR-1, AR-2 and AR-3 before comparing them with exact classical computation, and refines one rule (AR-3 to AR-3.1) after a falsification, the refinement being confirmed on a fresh seed; and (iv) tests a projection-aware rule for the regime of $x_{n+1} = \Pi(Tx_n)$ across dead-zone, clipping, lattice-rounding, state-gated and linear projections (PRD-1). In 4,197 nonlinear PRD-1 cases the rule made no false certified prediction: 3,159 verdicts are supported by analytical certificates, 819 are finite-horizon-consistent only (no contradiction in 200 exact steps, but no asymptotic certificate), and the rule abstained in 219. An independently written checker found no contradiction in 2,400 fresh cases; these are exact finite-horizon checks, not proofs of every asymptotic assertion. The central mathematical point is elementary but concrete: the spectrum of T alone does not determine the regime of $\Pi \circ T$; the projection, its thresholds and the starting state also matter. ESC-I does not overturn the original UNNS interpretation of complex numbers. Within the wider UNNS Substrate it is a specialised, exact-arithmetic extension of the complex-number programme; it is not an implementation of the canonical STRUC-I or STRUC-PERC-I admissibility predicates [11, 12], and it establishes no equivalence with them. It clarifies the conditions under which its structural mechanisms hold, and it does not establish that the algebra of complex numbers emerges uniquely from UNNS generative admissibility without its defining structure being assumed.

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1 Introduction

1.1 The original question

UNNS (Unbounded Nested Number Sequences) treats recursion as the basic generative act. Inside that programme, *Complex Numbers in the UNNS Substrate: Derived Convenience or Foundational Necessity?* [1] asks whether the substrate requires complex numbers or whether complex behaviour can be reconstructed from integer–algebraic lattices and cyclotomic embeddings. Its answer is that complex numbers are *not* foundational: oscillations, phases and spectra can be realised in rings of integers of cyclotomic fields, and $\mathbb{C}$ is best regarded as an efficient shorthand and completion. A companion note [2] makes the same point concretely, replacing i by a real quarter-turn R with $R^2 = -I$ and solving quadratics of negative discriminant by “echo cycles” in $\mathbb{R}^2$.

The present manuscript starts from a narrower and more testable corner of that programme: the *real recursive echo*. A complex multiplier acts on $\mathbb{R}^2$ as a real 2×2 matrix; iterating it generates an orbit (the echo). Two follow-up questions arise at once.

1. **Classification.** Given a real 2×2 matrix A and a starting state x_0 , can the long-time regime of $x_{n+1} = Ax_n$ be decided exactly, and how does that exact answer compare with what an observer sees after a finite number of steps at finite resolution?
2. **Admissibility under projection.** The original manuscript’s pipeline is not a pure linear recursion: each step is followed by an inlaying (rounding to a lattice) and a collapse (zeroing sub-threshold values). When does such a projected recursion $x_{n+1} = \Pi(Tx_n)$ keep the regime of T , and when does the projection *route* the orbit somewhere else?

1.2 How ESC-I arose

ESC-I was built to answer these questions with the smallest instrument that could do so honestly: exact rational arithmetic, deterministic seeds, rules written down and hashed *before* they are compared with exact computation, and failures kept in the record. The name is **Echo–Admissibility Correspondence, Chamber I**: it concerns the correspondence between recursive echo dynamics, exact mathematical classification, and structural admissibility within the UNNS Substrate. It is a chamber in the UNNS laboratory sense (a bounded, reproducible experimental environment), not a theory.

Development proceeded in four stages, each of which appears in this manuscript: (a) an initial chamber (exact classifier, echo classifier, perturbation challenges, an offline interactive page); (b) a theorem audit of the original §4 and §5 together with three frozen admissibility rules and their exact comparison; (c) a fresh-seed confirmation of a corrected quantiser clause (AR-3.1); and (d) the projection route-dependence experiment PRD-1, followed by an independently written checker.

Position within the UNNS research programme. ESC-I belongs to an established UNNS research architecture rather than introducing structural admissibility as a new concept. The canonical programme [9, 10] includes investigations of recursive invariants, refinement dynamics, generative selection, structural admissibility, vulnerability budgets, percolation, route closure [3, 14] and predictive exclusion [13]. In particular, STRUC-I v1.0.4 [11] evaluates perturbative admissibility of ordered numerical ladders, and STRUC-PERC-I v2.5.0 [12] investigates connectivity and fragmentation in full pairwise vulnerability graphs. ESC-I addresses a different mathematical object: the evolution of real recursive states under linear operators and subsequent projection. Its contribution is a specialised, exact-arithmetic development of the earlier complex-number programme within the wider UNNS approach to generative structure. It neither

replaces the canonical instruments nor asserts that their admissibility criteria are mathematically equivalent to its orbit-regime certificates.

1.3 From the Original Complex-Number Hypothesis to Structural Admissibility

The ESC-I programme is a continuation of the early UNNS complex-number research, not a departure from it. The central ideas of [1, 2] remain meaningful and are the starting point of everything below:

- complex multiplication admits an exact representation through real two-dimensional operators;
- cyclotomic structures provide an algebraic setting for finite phase recurrence;
- recursive evolution can express oscillatory and spectral behaviour without complex-valued state variables;
- projection and collapse can fundamentally alter the evolution of generated structures.

Each idea motivated a later piece of the investigation. The real representation led to the echo and to its exact classification (Section 3). Finite cyclotomic phases became the root-of-unity criterion for periodic orbits (Section 2.3). The role of projection and collapse led to the audit of the pipeline $\Pi C_\varepsilon T$, and the audit led to *explicit conditions*, an escape radius and a cone, under which the original mechanisms hold exactly (Section 4); the admissibility rules AR-1 to AR-3.1 and the projection experiment PRD-1 then tested those conditions against exact calculation (Sections 6–8). In this development the counterexamples serve to locate the boundaries of the mechanisms, and the refined statements are what the later experiments build on.

What ESC-I adds to the original programme is therefore (i) an exact, reproducible classification of real recursive echoes and of the gap between exact class and finite observation; (ii) proved conditions for phase inletting and for projected growth; (iii) frozen, falsifiable admissibility rules tested against exact computation; and (iv) a projection-aware rule, with an independent checker, for the regime of $\Pi \circ T$. It does not show that complex numbers are dispensable in every setting, and it does not show that UNNS uniquely generates their algebra without additional assumptions (Section 10).

1.4 Contributions and non-claims

Contributions. (1) Refined, proved statements, with explicit hypotheses, for the original phase-inletting and projected-stability claims (Theorems A and B). (2) A reproducible record in which one preregistered rule was falsified, refined and re-confirmed. (3) A projection-aware rule, validated in a stated domain against exact simulation and an independent checker, whose coverage is compared honestly with a projection-aware classical comparator. (4) A sharpened statement of what the complex-number question has and has not been shown to imply.

New and inherited. Items (1)–(4) are results of ESC-I itself. The methodological ingredients it uses, namely preregistration of rules, hashing, a falsification-first protocol and the vocabulary of admissibility, boundary and persistence, are inherited from the wider UNNS programme [9, 13] and are not claimed as ESC-I contributions.

Non-claims. Every rule here restates elementary dynamical-systems theory plus certificates. We claim no UNNS law beyond classical mathematics, no superiority over a classical analyst who is given the full definition of each projection, and no bearing on physical constants.

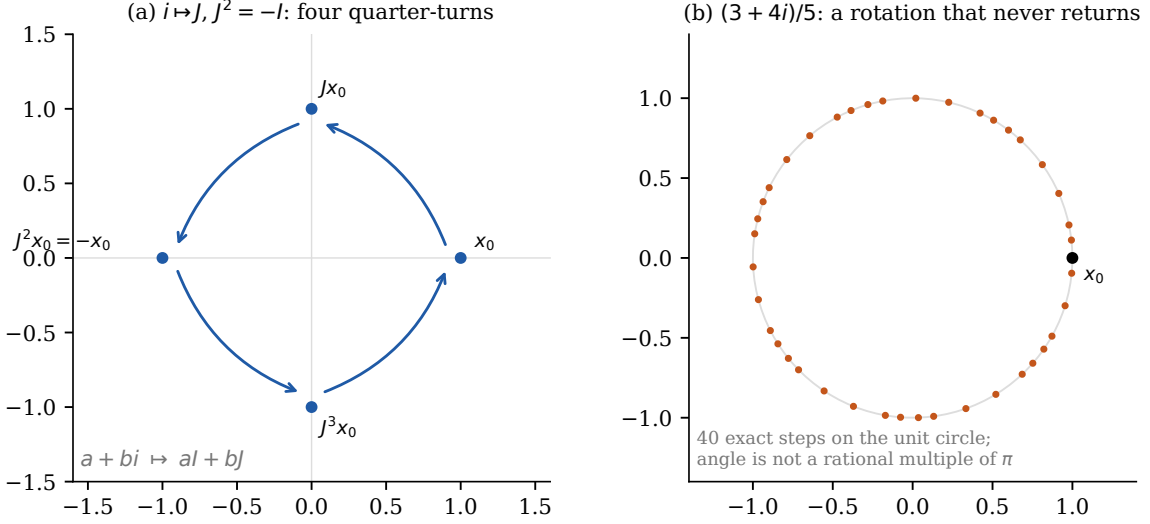


Figure 1: Complex multiplication as real rotation. (a) $i \mapsto J$: four successive quarter-turns return x_0 . (b) The Gaussian-rational multiplier $(3 + 4i)/5$ is also a real rotation of the plane, but its orbit never returns: the angle is not a rational multiple of π . Generated from exact rational orbits by `make_figures.py`.

1.5 Reading guide, provenance and conventions

Section 2 restores the complex-number background and the original manuscript’s content. Section 3 gives the recursive echo setting. Section 4 audits the original claims. Sections 5–8 describe the chamber, the rules, PRD-1 and the independent verification. Sections 9–12 interpret, revisit the complex-number question, state limitations and conclude. Appendices hold the proofs, counterexamples, reproducibility record and full numerical summaries.

Provenance. The audit in Section 4 is made directly against the text of the original manuscript [1], citing its §4 and §5 by number. All numerical results come from code in the ESC-I folder (`esc_i.py`, `admissibility.py`, `independent_check_ar31.py`, `independent_check_prd1.py`) and the reports in `output/`; every empirical statement is scoped to the generated cases listed. Rational numbers are exact throughout; $\rho(T)$ denotes the spectral radius. *Notation.* In STRUC-I [11] the symbol ρ denotes a structural pressure (an inversion-to-vulnerability ratio); the two quantities are unrelated by definition, and in this manuscript ρ always means the spectral radius. Evidence is kept in separate classes throughout: an exact ESC-I theorem, a fresh-seed computational confirmation, a finite-horizon consistency result, a STRUC-I statistical verdict and a STRUC-PERC-I connectivity classification are not interchangeable, and agreement between them is never assumed.

2 Complex Numbers in the UNNS Substrate

2.1 Complex multiplication as real rotation

The map $a + bi \mapsto aI + bJ$, with $J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, is an isomorphism of $\mathbb{C}$ onto the algebra $\mathcal{C} = \text{span}(I, J) \subset \mathbb{R}^{2 \times 2}$, because $J^2 = -I$. Multiplication by i is the quarter-turn J , and its orbit is the period-four echo $x_0 \rightarrow Jx_0 \rightarrow -x_0 \rightarrow -Jx_0 \rightarrow x_0$ (Figure 1a). Multiplication by $a + bi$ is the rotation–dilation $aI + bJ = \rho R_\theta$ with $\rho = \sqrt{a^2 + b^2}$. This is the sense in which the companion note [2] says that the “imaginary” part “is just planar rotation”.

2.2 Recursions with complex roots

For $a_{k+1} = \alpha a_k - \beta a_{k-1}$ with $\alpha, \beta \in \mathbb{Q}$ and $\alpha^2 < 4\beta$, the companion matrix $\begin{pmatrix} \alpha & -\beta \\ 1 & 0 \end{pmatrix}$ has trace α and determinant β . It is similar over $\mathbb{R}$ (not equal) to the rotation–dilation ρR_θ with $\rho = \sqrt{\beta}$ and $\cos \theta = \alpha/(2\sqrt{\beta})$ [1], Prop. 2. The real orbit therefore oscillates with a phase that no real eigenvalue exhibits; $\mathbb{C}$ is a compact encoding of the same dynamics.

2.3 Three kinds of phase

A distinction that the original manuscript uses but does not isolate turns out to organise the whole ESC-I investigation.

1. **Finite cyclotomic phases.** A recursion whose phase increment is a rational multiple of 2π is periodic and lives in $\mathbb{Z}[\zeta_n]$ for some n (the original Lemma 1 and Theorem 2). In $\mathbb{R}^{2 \times 2}$ over $\mathbb{Q}$ the finite-order elements of determinant one have order 1, 2, 3, 4 or 6 only, since the characteristic polynomial is cyclotomic of degree at most two. In the exact classifier this appears as the condition $\tau = \text{tr } A \in \{0, \pm 1\}$ for operators with $\det A = 1$ and negative discriminant to have finite order (order 4, 6, 3 respectively). *Operator order and orbit period are different notions.* A matrix has finite order if $A^n = I$; an individual orbit has period p if $x_{n+p} = x_n$. The first implies the second for every initial state, but not conversely: the shear $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ has infinite order, yet the orbit of its eigenvector $(1, 0)$ is a fixed point (period 1). The classifier’s class *periodic* describes orbits, and therefore contains both finite-order operators and special states of operators of infinite order.
2. **General complex rotations.** A Gaussian rational such as $(3+4i)/5$ has modulus one and lies in $\mathbb{Q}(i) = \mathbb{Q}(\zeta_4)$, yet it is *not* a root of unity. The rotation it induces on $\mathbb{R}^2$ is a genuine rotation (Figure 1b), with orbit dense on the circle, never periodic. Such a multiplier is representable inside a cyclotomic field as an *element*, but its *phase group* is infinite cyclic, so it is outside the finite-phase hypothesis of Lemma 1.
3. **Lattice embeddings with projection.** Inlaying $G : \mathbb{R}^2 \rightarrow \mathbb{Z}[i]$ and $E : \mathbb{R}^2 \rightarrow \mathbb{Z}[\omega]$ replace a state by a nearby lattice point, with error bounded uniformly (Prop. 1 of [1]: $\|x - G(x)\|_\infty \leq \frac{1}{2}$). This is what turns a linear recursion into a projected one and is the subject of Sections 6–7.

The exact classifier reports the second kind faithfully: the Pythagorean rotation $A = \begin{pmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{pmatrix}$ is classified *bounded, quasi-periodic* (Niven’s theorem excludes a rational angle), while the sixth-order companion of $x^2 - x + 1$ is *periodic*.

2.4 What the original manuscript claims

Table 1 lists the formal items of [1] and their status after the ESC-I audit; details are in Section 4, which cites the original §4 and §5 directly.

3 Recursive Echo Dynamics

3.1 Setting

Let $A \in \mathbb{Q}^{2 \times 2}$ and $x_0 \in \mathbb{Q}^2$. The *echo* is the orbit $x_{n+1} = Ax_n$. With $\tau = \text{tr } A$ and $\Delta = \det A$ the characteristic polynomial is $\lambda^2 - \tau\lambda + \Delta$. The 2×2 Jury criterion decides the stability of the eigenvalues in $\mathbb{Q}$: both lie strictly inside the unit disc iff $\Delta < 1$ and $|\tau| < 1 + \Delta$. Everything below that is needed in the boundary cases is decided by exact rational tests: Jordan structure, whether x_0 is an eigenvector, and whether $\tau \in \{0, \pm 1\}$ when $\Delta = 1$ and the discriminant is negative.

Item ([1])	Content	ESC-I status
Prop. 1	inlaying error bounds ($\leq \frac{1}{2}$ for G ; fixed constant for E)	accepted (G); E not examined
Lemma 1	finite-phase capture in $\mathbb{Z}[\zeta_n]$	accepted (classical); not tested
Prop. 2	real 2×2 rotation–dilation for complex eigenpairs	accepted, with “similar to”, not “equal to”
Lemma 2	$\ \mathcal{T}(x) - Tx\ \leq B$, $x_k =$ $T^k x_0 + \sum T^{k-1-j} \Delta_j$	accepted
Thm. 1	asymptotic spectral stability of the projected pipeline	holds under an added hypothesis; refined (Thm. B)
Prop. 3	$\rho(U_\theta T) = \rho(T)$ under constant phase inletting	holds for commuting or scaled-isometric T ; refined (Thm. A)
Rem. 2	adiabatic phases preserve the growth rate	not examined (conjectural)
Thm. 2, Cor. 1	$\mathbb{C}$ as a derived envelope of $\{\mathbb{Q}(\zeta_n)\}$	structural statement; see §10
§§6, 10	Hilbert embedding, physics interfaces	outside the scope of ESC-I

Table 1: Items of the original manuscript and their status after audit.

3.2 Exact classes

The exact classifier assigns each (A, x_0) one of four classes.

Class	Meaning
periodic	the orbit returns exactly (root-of-unity rotation, eigenvector at an eigenvalue of modulus one)
bounded	never returns, never leaves a ball (quasi-periodic rotation)
convergent	tends to 0, to a nonzero fixed point, or to a 2-cycle; includes finite-time collapse by a nilpotent A
divergent	unbounded: geometric growth, or linear growth through a Jordan block

Figure 2 shows one example of each class. The labels beneath the panels are the exact classifier’s output, computed rather than assigned.

3.3 State dependence

The class depends on the state as well as on the operator. For $T = \text{diag}(2, \frac{1}{2})$ the spectral radius is 2, yet the orbit from $x_0 = (0, 1)$ converges to the origin, while the orbit from $(\epsilon, 1)$ diverges for every $\epsilon \neq 0$ (Figure 3). The shear $(\begin{smallmatrix} 1 & 1 \\ 0 & 1 \end{smallmatrix})$ has $\rho = 1$ and is a fixed point from the eigenvector $(1, 0)$ but grows linearly from $(0, 1)$. Consequently the spectral radius alone is inconclusive: of the 15 designed examples of the initial chamber, the naive ρ -only test differs from the exact class in 9.

3.4 Observation at finite horizon and resolution

An observer sees only a finite orbit. The *echo classifier* of ESC-I uses the trajectory alone with horizon $N = 2000$ and tolerance 10^{-9} (period detection by a residual test, decay or growth by norm tails). Its disagreements with the exact class are not random; they are of two kinds (Figure 4).

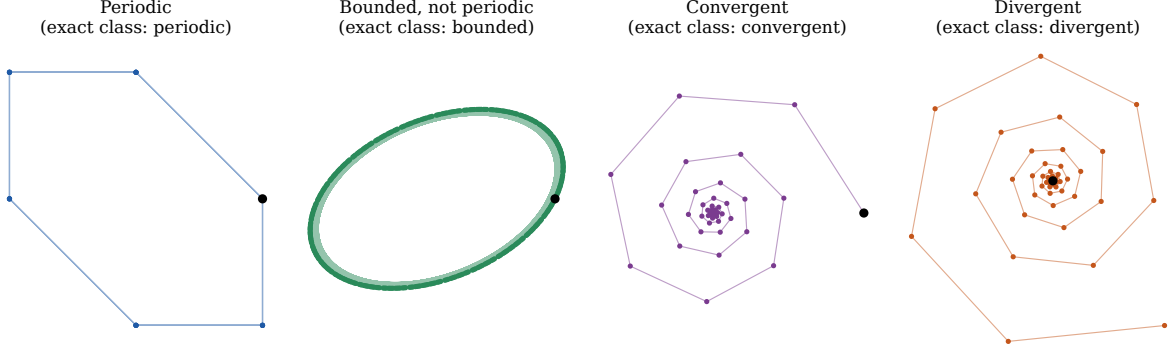


Figure 2: Four structural regimes of a real recursive echo, each classified exactly in $\mathbb{Q}$: periodic (order 6 companion of $x^2 - x + 1$), bounded non-periodic ($\tau = 3/2$, $\Delta = 1$), convergent (contracting spiral, $\rho = 0.9$) and divergent (expanding spiral, $\rho = 1.1$). The black dot is x_0 .

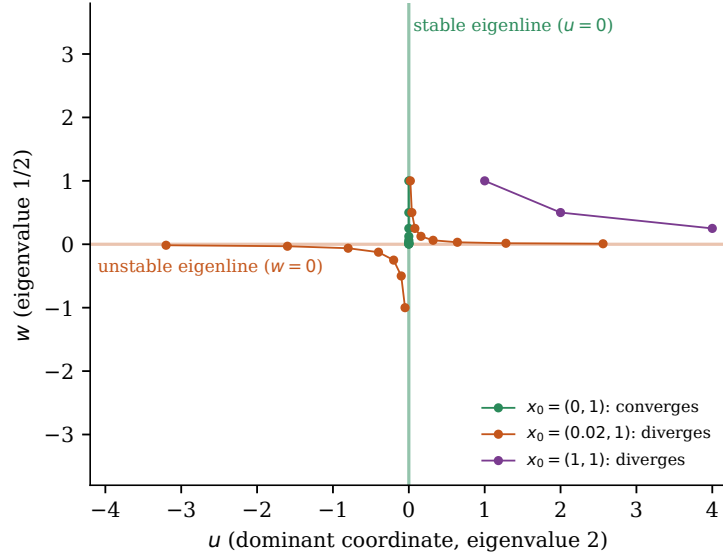


Figure 3: State-dependent admissibility for one saddle operator $T = \text{diag}(2, \frac{1}{2})$. The orbit started on the stable eigenline converges; orbits started arbitrarily close to it leave along the unstable eigenline. The same operator yields different structural regimes for different states.

- **Horizon trap.** $\rho = 1 + 10^{-7}$ is exactly divergent, but after 2000 steps the growth is $e^{2 \times 10^{-4}} \approx 1.0002$, so the echo reads *bounded*; about 1.4×10^8 steps would be needed.
- **Resolution trap.** $\tau = 1 + 10^{-12}$ gives an exactly quasi-periodic orbit, but the one-period residual $|x_n - x_{n-6}| \approx 4.5 \times 10^{-11}$ lies below the tolerance, so the echo reads *periodic*. The accumulated drift from x_0 does eventually exceed the tolerance, but the classifier tests only the per-period residual.

In the initial suite the echo and exact classes agree in 13 of 15 examples, and both disagreements are exactly these two designed traps.

4 Audit of the Original Mathematical Claims

The audit follows the text of [1]. For §4 the projected pipeline $\mathcal{T}(x) = \Pi C_\varepsilon(Tx)$ is written $x_{k+1} = F(x_k)$ with $F(x) - Tx = \Delta(x)$ and $\|\Delta(x)\| \leq \text{err}$, either an absolute bound h or a relative

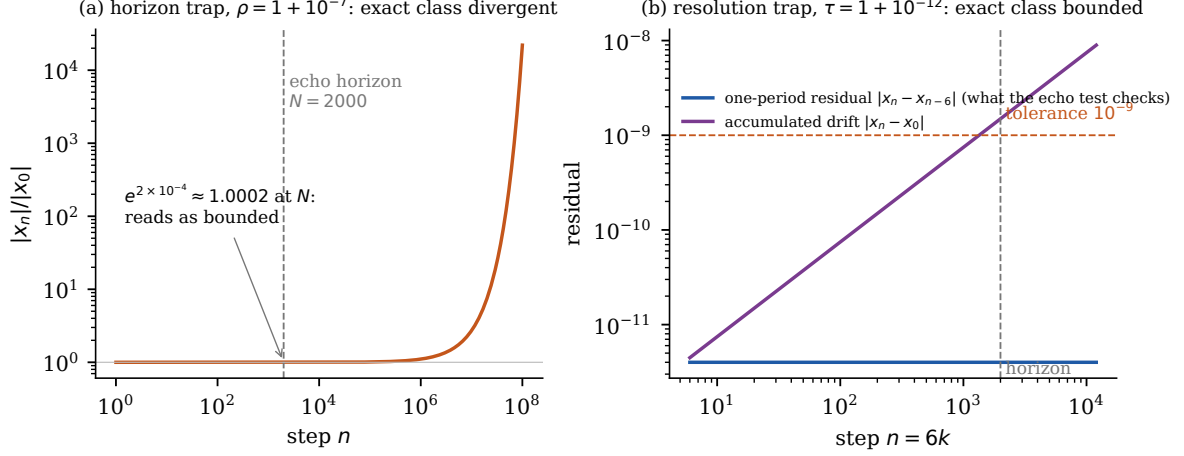


Figure 4: Observational traps. (a) Horizon trap: an exactly divergent orbit looks bounded within the echo horizon. (b) Resolution trap: an exactly quasi-periodic orbit looks period 6 because the per-period residual is below tolerance, although the accumulated drift would eventually exceed it.

bound $\eta\|x\|$. Lemma 2 of [1] is accepted in this form: rounding errors by at most $\frac{1}{2}$ per coordinate and zeroing sub-threshold values errors by at most ε . The statements below are proved or tested exactly against explicit examples; the status vocabulary is ESTABLISHED, CONDITIONAL, FALSE AS STATED, CONJECTURAL.

4.1 Phase inletting (§5 of [1])

The original statement (Prop. 3 of [1], §5) is that for constant $U_\theta = \text{diag}(e^{i\theta_1}, \dots, e^{i\theta_m})$ one has $\rho(U_\theta T) = \rho(T)$. This holds under conditions that the original statement leaves implicit; in full generality it needs an added hypothesis, as the following two readings of the object show.

Block reading, $m \geq 2$. Take $m = 2$, $\theta = (0, \pi)$, so $U = \text{diag}(1, -1)$, and $T = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$. Then $\rho(T) = 2$ but $UT = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$ is nilpotent, so $\rho(UT) = 0$, while $\rho(UTU^{-1}) = 2$ (similarity preserves ρ ; multiplication does not).

Single complex coordinate, real lift ($m = 1$). Here U_θ is the rotation R_θ and T a real 2×2 matrix. With T as above and $\theta = \pi/2$ one finds $R_{\pi/2}T = \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix}$, again nilpotent, so ρ drops from 2 to 0. The full answer is Theorem A.

Theorem A (Phase inletting under stated conditions). *Let $T = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathbb{R}^{2 \times 2}$.*

- (i) *For invertible U , UTU^{-1} (with $x_0 \mapsto Ux_0$) has the same orbit class, period and sub-type as T .*
- (ii) *If $UT = TU$ and all eigenvalues of U have modulus one, then $\rho(UT) = \rho(T)$. For $U = R_\theta$ with $\theta \notin \{0, \pi\}$, commutation is equivalent to $T \in \mathcal{C} = \text{span}(I, J)$, i.e. T is complex multiplication.*
- (iii) *$\det(R_\theta T) = \det T$ and $\text{tr}(R_\theta T) = (a + d) \cos \theta + (b - c) \sin \theta$.*
- (iv) *$\rho(R_\theta T)$ is independent of θ if and only if T is a scaled isometry, $T \in \mathcal{C} \cup \bar{\mathcal{C}}$ with $\bar{\mathcal{C}} = \{\begin{pmatrix} a & b \\ b & -a \end{pmatrix}\}$. Consequently $\sup_{U \in O(2)} \rho(UT) = \sigma_{\max}(T)$.*

The proof is in Appendix A. The statement most relevant to ESC-I is (iv): spectral invariance under phase inletting is not a property of every operator but characterises exactly the operators that are already complex-linear (or anti-linear). We return to this in Section 10.

4.2 Projected spectral stability (§4 of [1])

The original Theorem 1 of [1] (§4) asserts that for a diagonalizable T with simple dominant eigenvalue λ_* , the projected pipeline satisfies $\|x_{k+1}\|/\|x_k\| \rightarrow |\lambda_*|$ and $x_k/\|x_k\| \rightarrow v_*/\|v_*\|$. The proof outline reasons that the additive $O(1)$ error is negligible against $\|T^k x_0\| \asymp |\lambda_*|^k$. That step is where an added hypothesis is needed. Lemma 2 gives

$$x_k = T^k x_0 + \sum_{j=0}^{k-1} T^{k-1-j} \Delta_j, \quad \|\Delta_j\| \leq B,$$

and in the dominant direction the error sum is bounded by $B \sum_j |\lambda_*|^{k-1-j} = B(|\lambda_*|^k - 1)/(|\lambda_*| - 1)$, which is *of the same order* as $|\lambda_*|^k |u_0|$ rather than smaller. The error is negligible only when the initial dominant coordinate is large compared with $B/(|\lambda_*| - 1)$. The unrestricted statement therefore needs the hypothesis made explicit below, and the following examples, which satisfy every stated hypothesis of the original theorem, show why:

- $T = \text{diag}(2, \frac{1}{2})$ is diagonalizable with simple dominant eigenvalue. Let C_ε with $\varepsilon = 1$ zero sub-threshold values and $x_0 = (\frac{2}{5}, 1)$. Then $Tx_0 = (\frac{4}{5}, \frac{1}{2})$ is entirely sub-threshold, the projected orbit is 0 from step one, and $\|x_{k+1}\|/\|x_k\|$ does not tend to 2 (the orbit is exactly zero although $\rho(T) = 2$).
- With a dead-zone that zeroes only the dominant coordinate (the family DZ of Section 7), $x_0 = (\frac{2}{5}, 1)$ converges to 0 along the stable axis, a *structural transition from divergent to convergent caused by the projection*.
- With a relative error bound $\eta \geq |\lambda_*|$ the map $F \equiv 0$ is admissible; with a state on the stable eigenline $u_0 = 0$ even the unprojected orbit converges.
- For a complex dominant pair there is no dominant eigenline: the rate is robust to relative error, but an adversary can rotate the state by up to $\arcsin(\eta/r)$ per step, so the direction can drift without bound (up to 40.3 rad in 200 steps at $\eta = 0.3$, $r = 1.5$).

ESC-I therefore proves two refined statements, each with an explicit hypothesis.

Theorem B (Projected growth with escape radius). *Let $T = V \text{diag}(\lambda, \mu) V^{-1}$ with real λ, μ , $|\lambda| > 1$, $|\lambda| > |\mu|$. In eigen-coordinates $z = (u, w)$ with the sup norm let F satisfy $\|F(x) - Tx\| \leq \text{err}$.*

- (T4.1) (relative error) *If $\text{err} = \eta \|z\|$ with $\eta \leq (|\lambda| - |\mu|)/2$, and $|w_0| \leq |u_0|$, $u_0 \neq 0$, then for all n : $|w_n| \leq |u_n|$ and $|u_{n+1}|/|u_n| \in [|\lambda| - \eta, |\lambda| + \eta]$. The growth rate is therefore preserved up to η , and the invariant cone is $|w| \leq |u|$. In addition, $\limsup_n |w_n|/|u_n| \leq \eta/(|\lambda| - |\mu| - \eta)$. This last bound is asymptotic only: it need not hold for small n , since the hypotheses allow $|w_0|/|u_0| = 1$.*
- (T4.2) (absolute error h) *If $|u_0| > h/(|\lambda| - 1)$, then $|u_n|$ is strictly increasing, $|u_{n+1}|/|u_n| \rightarrow |\lambda|$, and the direction converges to the dominant eigenline. The radius $h/(|\lambda| - 1)$ is sharp for an adversarial projection: below it the projection can stall or erase u .*

Proofs are in Appendix A. The refined form of the original Theorem 1 is T4.2: its conclusion holds exactly under the added hypothesis $|u_0| > h/(|\lambda| - 1)$. Two consequences for the worked examples of [1] are worth recording. (a) In §7.1 (Fibonacci under Gaussian inlaying) the matrix and the initial integer vectors are integral, so the rounding is the identity and the example is exact; for a non-integer start T4.2 applies with $h = \frac{1}{2}$. (b) In §7.2 the claim $\lim \|x_{k+1}\|/\|x_k\| = \rho$ for $x_{k+1} = \Pi(C_\varepsilon(\rho R_\theta x_k))$ follows from the same inequality, since $|\Pi C_\varepsilon(\rho R_\theta x)| \geq \rho|x| - h$ in the Euclidean norm, but only for $\rho > 1$ and $|x_0| > h/(\rho - 1)$; for $\rho \leq 1$ the orbit can stay in a ball or collapse to 0 and the limit need not equal ρ . These two remarks are derived here and are not separately tested by the chamber.

4.3 Counterexample inventory and conjectures

Appendix B lists the exact counterexamples with their data. Two statements remain conjectural: **C1**, that $\rho(UT) = \rho(T)$ for all orthogonal U iff T is a scaled orthogonal matrix in $n \times n$ (proved here only for $n = 2$); **C2**, that T4.1 and T4.2 extend to Jordan blocks, to complex dominant pairs (for the rate only), and to cyclotomic-lattice projections with non-nearest-point II.

5 The ESC-I Experimental Chamber

5.1 Architecture

The chamber is deliberately small.

- **Exact classifier** (`esc.i.py`): Python `Fraction` arithmetic throughout; the Jury criterion, Jordan and eigenvector tests, the root-of-unity test, and the cross-product test for x_0 an eigenvector. Output: class and sub-type.
- **Echo classifier**: the trajectory-only observer of Section 3.4 (horizon $N = 2000$, tolerance 10^{-9}).
- **Challenges**: seeded perturbations of A (generic, area-preserving by composition with a unit-determinant shear) and of x_0 , to measure the largest perturbation size ϵ for which a class is held (“knife-edge” when it is 0).
- **Interactive chamber** (`chamber.html`): an offline page with a JavaScript port of the engine and the same seeded generator, parity-checked against Python.
- **Admissibility module** (`admissibility.py`): theorem audit, frozen rules, exact comparison, the AR-3.1 confirmation and PRD-1, in one run with three plus two outputs.

Eleven automated tests exercise the chamber: exhaustive classification of all 625 integer matrices with entries in $[-2, 2]$ against an independent exact orbit computation, agreement of $\rho \leq 1$ with floating-point eigenvalues on 3000 random rational matrices, the phase-inletting counterexample, determinism of the seeded challenge, a source check that rule functions never call the exact classifier, and full-run determinism with the frozen-before-compared ordering.

5.2 Initial findings

On the initial 15-example suite the echo agrees with the exact class in 13 cases, the two exceptions being the designed traps of Section 3.4; the ρ -only test differs from the exact class in 9; and the persistence profile is classical: periodic orbits are knife-edge under generic and under area-preserving perturbation (hold $\epsilon = 0$) while non-escaping behaviour survives area-preserving perturbation at every ϵ tested; marginal $\rho = 1$ cases show survival fractions that do not tend to 100% as $\epsilon \rightarrow 0$, which is codimension-one boundary geometry. None of this is a UNNS-specific prediction. The exact classifier is classical linear algebra, so the initial chamber provides a measuring instrument and not a predictive advantage. That limitation is what motivated the rule layer of Section 6.

6 Structural Admissibility Rules

6.1 Protocol

The rules AR-1, AR-2 and AR-3 turn the audited mathematics into *predicates that make a claim or abstain*. Their scientific value lies in the protocol, which is the same for each rule:

1. **Freeze.** The rule text and every test parameter are written and hashed (SHA-256) before any comparison.
2. **Predict.** Rule functions produce verdicts; the module checks by source inspection that they never call the exact classifier.
3. **Hash predictions** before the exact comparison.
4. **Compare** with exact classical computation. Outcomes are AGREE, DISAGREE, ABSTAIN (the exact computation resolves the case, the rule does not) and UNCONFIRMABLE. A DISAGREE means a falsified rule or an implementation error, and each is listed individually. Disagreements are recorded as data and do not stop the run.

Post-hoc amendments are allowed only if labelled exploratory and then confirmed on a fresh seed. The frozen rules hash is `c32fa4fd...ec19b54` (seed 20261013; full value in Appendix C).

6.2 AR-1: route preservation under phase and similarity transformations

Content. (a) Conjugation $T \mapsto UTU^{-1}$ with $x_0 \mapsto Ux_0$: ADMIT, class, period and sub-type preserved. (b) The left-rotation family $T \mapsto R_\theta T$: ρ -INVARIANT for scaled isometries; otherwise compare the exact regime of ρ at the extreme values $|\tau| = 0$ and $|\tau| = m$, $m^2 = (a+d)^2 + (b-c)^2$ of the trace sweep, answering REGIME-PRESERVED, REGIME-CAN-CHANGE or abstain. (c) General left multiplication $T \mapsto UT$: abstain (a negative control). **Role.** It encodes Theorem A: phase inletting is harmless exactly for conjugations and for scaled isometries.

6.3 AR-2: boundary sensitivity

Content. For diagonalizable T with $\rho \neq 1$ the certified margin is $\varepsilon_{\text{cert}} = \min(|\rho - 1|, \text{gap}/2 \text{ if } \rho > 1)/\kappa_2(V)$ (a Bauer–Fike type bound [5]); every real perturbation E with $\|E\|_2 < \varepsilon_{\text{cert}}$ keeps the regime $\rho \leq 1$. At $\rho = 1$ the case is KNIFE-EDGE: for every $\epsilon > 0$ some perturbation of norm at most $\epsilon\|T\|$ changes the regime. Non-diagonalizable T : UNCERTIFIED. **Role.** It says where the regime is robust, and quantifies how conservative the certificate is through a stated, falsifiable sharpness hypothesis ($K = 10$).

6.4 AR-3: projection persistence

Content. For $x_{n+1} = \Pi(Tx_n)$ with T in the spectral setting of Theorem B: an absolute error bound h guarantees ESCAPE whenever $|u_0| > h/(|\lambda| - 1)$ (below this threshold escape is not guaranteed, although particular admissible projections may still permit it); relative error gives the CONE verdict under T4.1; dead-zone and nearest-rounding projections have their own escape/no-escape conditions; and the k -bit relative quantiser converts 2^{-k} into an eigen-coordinate η . **Role.** It is the rule that predicts, from (T, Π, x_0) , whether a projected orbit keeps the dominant regime.

6.5 Results of the frozen comparison

The AR-3 falsification and its refinement. The frozen quantiser clause predicted an eigen-coordinate relative error of $\kappa_\infty(V)2^{-k}$. The comparison falsified it in 92 of 540 cases. The cause was an error in my derivation: a k -bit quantiser rounds Ty , so its error is relative to $|Ty|$, not $|y|$, and the correct conversion is $\eta_{\text{eig}} = \kappa_\infty(V)|\lambda|2^{-k}$. The corrected clause was explored post hoc on the same cases (234 agree, 306 abstain, 0 disagree, labelled exploratory) and then frozen as AR-3.1 *before* a fresh confirmation set was generated.

AR-3.1 confirmation. Seed 20261113: 60 random systems $\times$ bits $\{4, 8, 12\} \times 15$ states, plus boundary families with η_{eig} set 1% above, at, and 1% below the cone limit. Result: 3600

Rule	Test	Agree	Disagree	Abstain
AR-1a	conjugation preserves class	400	0	0
AR-1b	rotation family, 625 matrices	521	0	104
AR-1c	negative control (general U)	—	—	300 (28 changed class)
AR-2	safety, 12 000 perturbations at $0.98 \varepsilon_{\text{cert}}$	12 000	0	—
AR-2	knife-edge, $\rho = 1$	116	0	—
AR-2	sharpness, $K = 10$	114	0	6 unconfirmed
AR-3	projection persistence, 3240 cases	1986	92	1162

Table 2: Frozen comparison against exact computation. All 92 AR-3 disagreements are in one clause (relative k -bit quantiser, REL_QUANT: 207 agree, 92 disagree, 241 abstain of 540). Safety and knife-edge rows count perturbation and scaling checks; the six unconfirmed sharpness cases are cases for which the fixed direction search found no witness, not refutations (an exploratory wider search found witnesses at ratios between 10.8 and 18.8 for five of them).

Stratum	Cases	Agree	Abstain	Abstain but cone holds
random systems	2700	1668	1032	878
boundary, just outside (-1%)	300	0	300	240
boundary, at limit	300	240	60	1
boundary, just inside ($+1\%$)	300	240	60	1
total	3600	2148	1452	1120

Table 3: AR-3.1 confirmation by stratum. “Cone holds” means that the simulated orbit actually stayed in the cone although the rule’s sufficient condition failed.

cases, 2148 agree, 0 disagree, 1452 abstain (Table 3); zero violations of the per-step error bound; the frozen original conversion (without $|\lambda|$) is violated in 1359 cases, showing that the factor is necessary; the largest observed/bound ratio is 0.748. The amended source hash equals the hash recorded in the earlier run (2cf5d68a...13ff3f1; full value in Appendix C).

Closure wording. The corrected conversion holds under the rule’s stated conditions, with a sufficient but conservative cone condition. Table 3 quantifies the conservatism: in 240 of 300 just-outside cases the cone held anyway. The condition was *not* adjusted using this observation. The original 92 disagreements remain in `admissibility_report.json`.

What the rules are. AR-1 and AR-3 restate theorems proved in Section 4 plus certificates; AR-2 restates a standard perturbation bound. Agreement is therefore expected, and a disagreement indicates an implementation error or a wrong theorem, as happened once. The informative quantities are coverage (abstentions that exact computation resolves), the sharpness ratio, and the negative control. No advantage over the classical baseline is claimed.

7 Projection Route Dependence (PRD-1)

7.1 Question and domain

Two systems with identical unprojected data (T, x_0) can have different projected asymptotic behaviour. PRD-1 asks whether a rule that examines the *structure of the projection* predicts the projected regime on unseen systems and projections, in cases where a classical method that knows only the projection’s error bound cannot.

The domain is a stated restriction: the *spectral frame*, $T = \text{diag}(\lambda, \mu)$ with rational λ, μ , $|\lambda| > 1$, $|\lambda| > |\mu|$, state $z = (u, w)$, $u_0 \neq 0$, $\Pi(0) = 0$, and Π applied to $y = Tz$. Projections written in another frame need a frame conversion and are out of scope.

Definition 1 (Projection families). • **DZ**(θ), dead-zone: $u \mapsto 0$ if $|u| < \theta$, otherwise identity;

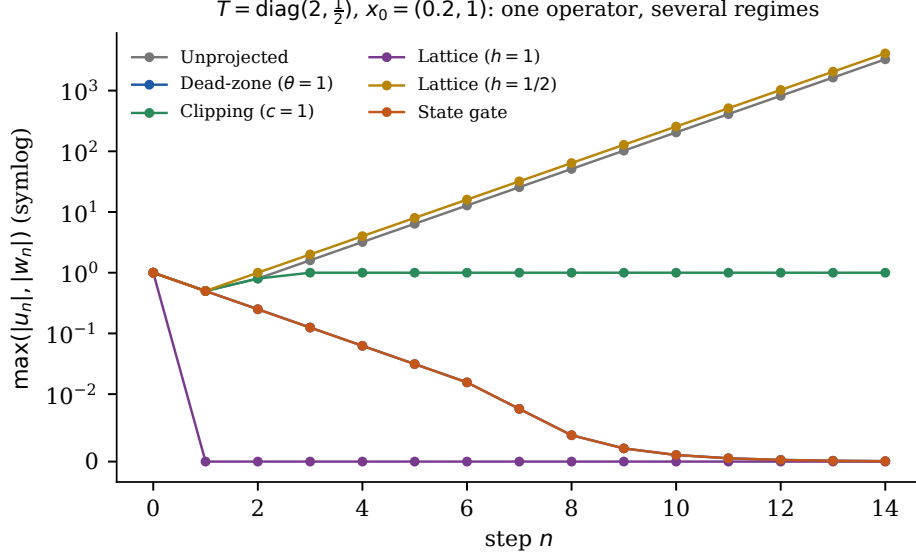


Figure 5: Projection-dependent structural routing. The operator $T = \text{diag}(2, \frac{1}{2})$ and the state $x_0 = (0.2, 1)$ are fixed. Unprojected, and under lattice rounding with $h = \frac{1}{2}$, the orbit diverges; the dead-zone ($\theta = 1$) and the state gate send it to 0 along the stable axis; clipping ($c = 1$) makes it bounded; lattice rounding with $h = 1$ collapses it to 0 in one step. Exact rational orbits (symlog axis).

w untouched.

- **CLIP**(c), saturation: each coordinate clamped to $[-c, c]$.
- **LAT**(h), lattice rounding: each coordinate rounded to the nearest multiple of h (ties to even).
- **GATE**, state-gated zeroing: $u \mapsto 0$ if $|u| \leq |w|$, otherwise identity.
- **LINP**(a), **LINQ**(b), linear kernel projections (control): $\text{LINP}(y) = (0, ay_1 + y_2)$, $\text{LINQ}(y) = (y_1 + by_2, 0)$. The orbit is the linear recursion with matrix ΠT .

Figure 5 shows the central phenomenon on one operator and one state.

7.2 Verdicts, rule R and comparator C

Verdicts: DIVERGENT ($|z_n| \rightarrow \infty$), COLLAPSE ($z_n = 0$ from some step), CONVERGENT ($z_n \rightarrow 0$), BOUNDED(B) ($\|z_n\|_\infty \leq B$ for $n \geq 1$), ABSTAIN, and, for linear Π only, LINEAR-REDUCTION (the rule recognises the case as reducible to the classical analysis of ΠT and claims no regime).

Rule R uses the structure of Π and may neither simulate nor call the exact classifier. Write $\alpha = |\lambda|$, $\beta = |\mu|$.

- **DZ**: if $\alpha|u_0| \geq \theta$, DIVERGENT. Otherwise u is erased at step 1 and stays 0; $w_1 = \mu w_0$. If $w_1 = 0$: COLLAPSE; $\beta < 1$: CONVERGENT; $\beta = 1$: BOUNDED($|w_0|$); $\beta > 1$: DIVERGENT.
- **CLIP**: BOUNDED(c).
- **LAT**: DIVERGENT if $|u_0| > (h/2)/(\alpha - 1)$, or if $\beta > 1$ and $|w_0| > (h/2)/(\beta - 1)$; COLLAPSE if $\alpha|u_0| < h/2$ and $\beta|w_0| < h/2$; BOUNDED($\beta|w_0| + h/2$) if $\alpha|u_0| < h/2$ and $\beta \leq 1$; otherwise abstain.
- **GATE**: if $\alpha|u_0| > \beta|w_0|$, DIVERGENT (the ratio $|w|/|u|$ shrinks by β/α , so u is never gated). Otherwise u is erased at step 1 and stays 0, with the same β branches as DZ.

Family	Cases	Rule R (predictions issued)		Comparator C	Contradicted
		all	analytical	certified	R, C
Dead-zone	1560	1560 (100%)	1441 (92.4%)	871 (55.8%)	0, 0
Clipping	1080	1080 (100%)	738 (68.3%)	0 (0%)	0, 0
Lattice	1170	951 (81.3%)	951 (81.3%)	907 (77.5%)	0, 0
State gate	387	387 (100%)	29 (7.5%)	0 (0%)	0, 0
All nonlinear	4197	3978 (94.8%)	3159 (75.3%)	1778 (42.4%)	0, 0

Table 4: PRD-1 nonlinear results. Percentages use the family’s own case count as denominator (the last row uses all 4197 nonlinear cases). “All” counts predictions issued, that is, analytically supported plus finite-horizon-consistent; “analytical” counts only predictions with an analytical certificate; the comparator column counts C’s certified predictions. R abstains on 219 lattice cases (18.7% of that family) and nowhere else; C abstains on 2419 (57.6% of all nonlinear cases). 819 of R’s 3978 predictions (19.5% of all cases) are finite-horizon only (119 dead-zone, 342 clipping, 358 state gate).

- LINP, LINQ: LINEAR-REDUCTION.

Comparator C is a *projection-aware classical method*, not the unprojected classifier. It knows T and the projection’s error bound: DZ has absolute bound θ , LAT has $h/2$, while CLIP and GATE have no finite absolute bound (and GATE’s relative bound $\eta = \alpha$ violates $\eta \leq (\alpha - \beta)/2$), so C abstains on them. For DZ and LAT it applies the escape-radius theorem T4.2; for linear Π it applies the exact classifier to the matrix ΠT . The unprojected classifier, which returns *divergent* for every system in the test set, serves only as the route-dependence baseline.

7.3 Protocol and evidence levels

The experiment’s rules, families, parameters, evidence definitions and success criteria were frozen and hashed before any case was generated (seed 20261213; rules hash `3bf6d366...dab07d`); the predictions of R and C were hashed before simulation (`fbe397db...c698a642`; full values in Appendix C). The test set has 30 systems with $\lambda = \pm m/4$, $m \in 5, \dots, 20$, $|\mu|$ from a fixed grid below $|\lambda|$, and for every family and parameter 12 random states together with exact tie states; the totals are 4197 nonlinear and 1440 linear-control cases, 5637 in all.

Finite horizon versus asymptotic claim. Ground truth is exact rational simulation of $N = 200$ steps. Exact simulation removes floating-point uncertainty but does not by itself establish infinite-time behaviour. A verdict is therefore CONFIRMED-ASYMPTOTIC only if an analytical certificate supports it: an exact hit of 0 ($\Pi(0) = 0$ makes 0 absorbing, hence COLLAPSE); an exact repetition of a state (deterministic autonomous recursion, hence periodic and BOUNDED); or an a posteriori escape-radius certificate (DZ and LAT, the families with a finite absolute error bound). Otherwise a verdict that survives 200 steps without contradiction is labelled CONSISTENT-FINITE-HORIZON. Contradiction criteria are fixed constants chosen so that every divergent or convergent system clears them by orders of magnitude at $N = 200$.

Success criteria (stated in advance). (1) Zero contradicted certified predictions for R and for C. (2) Coverage reported per family, with cases certified by R only, C only and both. (3) For cases where the unprojected classifier is wrong about the projected regime, the number certified correctly by R and by C. (4) The linear-recognition check (exact additivity and homogeneity of Π on seeded triples) passes for all families, and the control orbit equals the matrix recursion ΠT exactly. (5) Computational cost is reported separately from correctness.

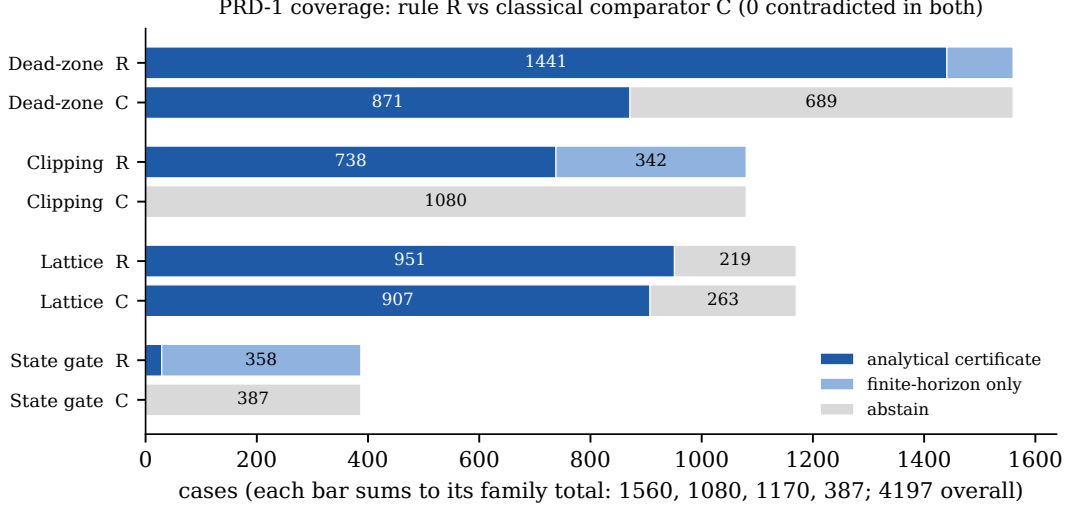


Figure 6: PRD-1 rule coverage by projection family from the recorded report. Dark bars: analytical certificates; light bars: finite-horizon-consistent only (no asymptotic certificate); grey: abstentions. Each bar sums to the family’s case count (1560, 1080, 1170, 387; 4197 overall). Neither rule contradicts exact simulation anywhere. C’s abstentions on clipping and state-gating reflect the information it is given (an error bound only).

7.4 Findings

Contradictions. Rule R was contradicted in 0 of 4197 nonlinear cases, including all exact tie states. The comparator C was contradicted in 0 nonlinear cases.

Coverage. R issued 3978 predictions, of which 3159 were analytically supported and 819 finite-horizon-consistent; C issued 1778 certified predictions; the overlap is R-only 2200, C-only 0, both 1778, neither 219. R’s extra coverage comes from clipping and state-gating, where C has no usable error bound, and from dead-zone cases whose zero-erased dominant coordinate is predicted exactly. On lattice rounding, where both rely on an error bound, they are near parity (951 against 907).

Route dependence. The unprojected classifier is wrong about the projected regime in 2061 cases. In the 1578 nonlinear cases where the projected regime is not divergent, R’s prediction is correct in 1445 and C in 0 (C cannot reach COLLAPSE or CONVERGENT claims from an error bound).

Cost. R evaluates a closed-form condition per case with no orbit. The simulation used for scoring cost 200 exact steps per case. This is a statement about computation, not about logical correctness, and it does not make R faster than a classical analyst who writes down the same condition.

7.5 The linear control and a comparator error

The linear projections LINP and LINQ are reduced to the matrix ΠT and classified by the exact classifier. The recognition check identified the linear families as linear and the nonlinear ones as nonlinear, and the control orbit equals the matrix recursion in all 1440 cases. The control is there so that established linear algebra is not presented as new.

First-run record. In the first run, rule R had 0 contradictions but the *comparator* had 144 contradicted verdicts on the LINP control, so the frozen all-criteria condition failed. The cause was a mapping error in my comparator code: the exact classifier’s “convergent” includes convergence to a nonzero fixed point or a 2-cycle, which I had mapped to CONVERGENT (limit

Family	Cases	Claims checked	Abstained	Contradictions
Dead-zone	400	400	0	0
Clipping	400	400	0	0
Lattice rounding	400	325	75	0
State gate	400	400	0	0
Linear P (control)	400	400 closed-form	0	0
Linear Q (control)	400	400 closed-form	0	0
Total	2400	1525 nonlinear + 800 linear	75	0

Table 5: Independent PRD-1 check, seed 20261014. The run recorded in the project folder as `RUN_RESULT.json` is identical to the stored historical summary, and an identical result was obtained by re-running the script in the cloud workspace.

0). For example $\lambda = -2, \mu = -1, a = \frac{1}{2}$ gives $\Pi T = \begin{pmatrix} 0 & 0 \\ -1 & -1 \end{pmatrix}$, whose orbit converges to a 2-cycle, not to 0. Rule R was unaffected.

Amendment. The correction (nonzero limits and 2-cycles map to BOUNDED) is a *post-hoc, exploratory* amendment; the first-run numbers are retained in the report. With it the control has 0 contradictions on the same seed. On a fresh seed (20261790, run after the amendment, hence exploratory and not preregistered) the first-run mapping again produces 48 control contradictions and the amended mapping 0, with R again contradicted in 0 of 4197 cases (certified 3989, of which 2979 asymptotic and 1010 finite-horizon, abstained 208; C certified 1838). Accordingly the accurate summary is: *zero contradicted predictions for R; the preregistered all-criteria condition held only after a documented post-hoc comparator correction.*

7.6 What PRD-1 supports

The strongest justified claim is: within the stated spectral-frame domain, rule R produced 3159 analytically supported predictions, 819 further predictions consistent with 200 exact iterations, and 219 abstentions, with no observed contradiction against exact simulation or the independent checker (Section 8). It makes its predictions without step-by-step trajectory simulation. That is not a claim of superiority over classical analysis: a classical analyst who reads the same projection definition can derive the same conditions, and the rule itself is elementary.

8 Independent Verification

Independence is layered, and each layer establishes something different.

Layer 1: stored-record consistency. Re-running `admissibility.py` reproduces the stored reports exactly (determinism of seeds and hashes). This shows the record has not been altered, not that the mathematics is right.

Layer 2: AR-3.1 independent bound check (`independent_check_ar31.py`). A standard-library script with its own random generator, quantiser and exact arithmetic, importing nothing from ESC-I, tests the converted bound $\kappa_\infty(V)|\lambda|2^{-k}$ itself. Run here with seed 12345 on 400 systems (16000 steps): 0 violations of the corrected bound, 1341 of the original bound, largest observed/bound ratio 0.774. This is evidence for the bound, not merely for consistency.

Layer 3: PRD-1 independent checker (`independent_check_prd1.py`, folder `PRD1_VERIFY`). A separately written standalone script uses Python’s `random.Random(20261014)`, `fractions.Fraction`, its own implementation of each projection and its own predicted classes, and does not import `admissibility.py` or `esc_i.py`. It draws a fresh sample of 400 cases per family (2400 in all) and checks nonlinear predictions by exact rational iteration over 200 steps and the linear controls against directly derived closed-form recurrences over 12 steps. The

accompanying audit also re-derives each rule by elementary arguments: for the dead-zone, if $|\lambda u_0| \geq \theta$ the expanding coordinate is never removed; for clipping, Π maps into $[-c, c]^2$, so every later iterate is bounded; for lattice rounding the reverse triangle inequality gives strict growth beyond $h/[2(|\lambda| - 1)]$; for the gate, $|\lambda u|$ against $|\mu w|$ at step one decides, because dominance can only increase afterward; and the linear controls evolve as $(0, \mu^{n-1}(a\lambda u_0 + \mu w_0))$ and $(\lambda^{n-1}(\lambda u_0 + b\mu w_0), 0)$ respectively.

Qualifications. (1) The checker draws a distinct sample and seed; it does not reconstruct the exact 5637-case sample, so the case counts, hashes and timestamp ordering of the main run remain archive records and not independently reproduced facts. (2) It re-implements the rules independently of the code but not of their author; fully independent verification also needs a reviewer of the mathematics, or a separately conceived checker. (3) Exact finite-horizon reproduction is not a proof of every asymptotic assertion: the 819 finite-horizon verdicts remain explicitly finite-horizon, and “confirmed-asymptotic” is an evidence label resting on selected analytic certificates, not a proof-checker output. (4) The strict first-run success criterion remains false as frozen; the amended criterion is a separate post-hoc result and history must not be rewritten as a frozen pass. (5) The comparator’s smaller coverage reflects its information, not a limit of classical mathematics.

Verdict. Independent corroboration passes within the stated diagonal spectral-frame families, with these qualifications: no contradiction in 1525 nonlinear predictions or 800 linear reductions on 2400 fresh cases. It is not a universal projection-persistence theorem, and it is not a verified computational advantage over equally informed classical analysis.

9 Structural Interpretation

9.1 Route and boundary

The UNNS route-and-boundary viewpoint holds that what a generative process produces depends on the route it takes and the boundaries it meets, and not only on the generating rule’s local data (compare the route-closure programme [3]). ESC-I realises a concrete, minimal instance. In $x_{n+1} = \Pi(Tx_n)$ the pair (T, x_0) fixes an unprojected route, but the observed regime is a property of the *whole generative process* (T, Π, x_0) : the threshold of a dead-zone is a boundary the orbit either crosses or does not (the escape radius $h/(|\lambda| - 1)$ and the dead-zone condition $|\lambda u_0| \geq \theta$ are exactly such boundaries), and the stable eigenline is a route that exists only for particular states.

9.2 Relationship to canonical admissibility and percolation

Three structurally related but mathematically distinct levels must be separated. First, perturbative ladder admissibility in STRUC-I [11] concerns whether rank inversions $\text{inv}(P_\varepsilon; L)$ remain within a vulnerability-derived budget $\nu(V_\varepsilon(L))$. Second, STRUC-PERC-I [12] builds the full pairwise graph with edges $(i, j) \in E_\kappa \iff |\Delta_i - \Delta_j| \leq \varepsilon(\kappa)$ and studies connectivity across a scale interval; its formal interpretation distinguishes established results for the specified non-percolation and violation setting from the still-open converse from percolation to admissibility, which requires independent STRUC-I confirmation. Third, ESC-I studies the dynamics of a state under the projected recursion $x_{n+1} = \Pi(Tx_n)$; its sufficient conditions concern escape, boundedness, invariant cones and projection-dependent route changes.

These instruments share a vocabulary of generation, vulnerability, boundaries and persistence, but their predicates act on different spaces and cannot be interchanged without a defined transformation and a demonstrated preservation theorem. No such cross-instrument equivalence theorem is established here, and ESC-I does not close the open percolation-to-admissibility conjecture.

9.3 What is established within UNNS

Within the stated domain and evidence scope, ESC-I establishes:

- an exact, reproducible classification of real 2×2 echoes, with the observational gap between exact class and finite-horizon reading quantified by identified traps;
- corrected, proved conditions replacing two unrestricted claims of the original manuscript;
- that admissibility rules can be written, frozen, falsified, repaired and re-confirmed under an explicit protocol;
- that a projection-aware rule predicts the regime of $\Pi \circ T$ for four projection families in the spectral frame, with the evidence levels, abstentions and finite-horizon share reported.

9.4 What remains classical mathematics

Every proved statement is elementary: the Jury criterion, triangle-inequality escape radii, the Bauer–Fike type margin, Niven’s theorem, and the structure of 2×2 matrices. The route-dependence statement is the familiar fact that a nonlinear map’s behaviour cannot in general be read from its linearisation outside the region where the linearisation applies. ESC-I contributes a controlled, falsification-oriented record and sharpened hypotheses, not a new law.

10 The Complex-Number Question Revisited

The original manuscript asks whether complex numbers are a derived convenience or a foundational necessity, and answers that they are derived: representable in cyclotomic rings, with $\mathbb{C}$ a convenient completion. ESC-I bears on this in three ways, and it is important to separate what it shows from what it does not.

(1) The representation is recovered. For real 2×2 recursions, complex multiplication is the algebra $\mathcal{C} = \text{span}(I, J)$, and every complex-eigenpair recursion is similar over $\mathbb{R}$ to a rotation–dilation. All of the chamber’s exact classification, including the root-of-unity criterion $\tau \in \{0, \pm 1\}$, runs in real rational arithmetic with no appeal to $\mathbb{C}$.

(2) The three kinds of phase behave differently. The exact classifier separates finite cyclotomic phases (periodic), general Gaussian-rational rotations such as $(3 + 4i)/5$ (quasi-periodic, never returning), and projected lattice dynamics (where the projection can change the regime). A theory that treats “cyclotomic” as meaning “periodic” would misclassify the middle case; the original manuscript’s Lemma 1 is correctly restricted to finite phases, but its Corollary on pipelines “built from algebraic operators and cyclotomic inlayings” should be read with that restriction.

(3) Complex structure is singled out by an invariance property. Theorem A(iv) shows that phase inletting is spectrally inert, $\rho(R_\theta T)$ independent of θ , exactly for $T \in \mathcal{C} \cup \overline{\mathcal{C}}$, and that commutation with all rotations holds exactly for $T \in \mathcal{C}$. In other words, among real 2×2 operators, the complex-linear ones are characterised by invariance under the phase operation. This is a *derived characterisation* of complex multiplication within the real recursive setting.

What the original programme retains. ESC-I does not overturn the original UNNS interpretation of complex numbers. The exact real representation of complex multiplication, the cyclotomic setting for finite phase recurrence, the expression of oscillation and spectrum through real recursion, and the effect of projection and collapse on generated structure all remain meaningful. What the audit changes is the set of conditions under which certain stronger statements hold, and those conditions are what the later experiments tested.

What has not been shown. The characterisation in (3) presupposes the rotation family R_θ , that is, the structure J with $J^2 = -I$ is already present in the definition of the phase

operator. ESC-I therefore does *not* demonstrate that the algebra of complex numbers emerges uniquely from UNNS generative admissibility without its defining structure being assumed. It shows that complex structure is representable and characterisable inside real recursion; whether it can be *generated* by admissibility alone, with no rotation operator supplied, is an open question and is left open here. It is the complex-number instance of the generative-selection and route-closure questions of the wider programme [3, 14]: resolving it requires an independently specified generative grammar and a selection theorem, not further confirmation of the present recurrence experiments.

11 Limitations and Open Questions

1. **Domain.** All PRD-1 and AR-3 statements are for 2×2 real diagonalizable T in the spectral frame with $|\lambda| > |\mu|$, $|\lambda| > 1$. Jordan blocks, complex dominant pairs (rate only) and $n \times n$ operators are open (conjectures C1, C2).
2. **Finite horizon.** 819 PRD-1 verdicts rest on 200 exact steps without an infinite-time certificate. These are labelled as such throughout.
3. **Conservative conditions.** AR-3.1's cone condition is sufficient but conservative (Table 3); AR-2's margin is conservative by the sharpness ratio, which the fixed search could not confirm in 6 of 120 cases.
4. **Verification.** The independent PRD-1 checker corroborates a restricted family set on a distinct sample; it is an independent implementation, not an independently conceived theory, and not a proof checker.
5. **Comparator information.** C was given an error bound only; a classical analyst given the definition of each projection could match R's coverage.
6. **Protocol history.** The AR-3 clause failed once, and the PRD-1 comparator was mis-mapped once; both corrections are post-hoc and recorded as such.
7. **Scope of the original manuscript.** The Hilbert-space, thermodynamic and gauge-phase parts of [1] are outside the scope of ESC-I.
8. **Cross-instrument scope.** ESC-I does not establish a correspondence between projected orbit regimes and the classifications of STRUC-I or STRUC-PERC-I. Such a correspondence would require an explicit, reproducible map from recursive orbit data to ordered ladders, a stated preservation criterion, and independent tests against both canonical engines. The projection-route findings therefore are not additional validation of the Universal Structural Law [10], not a proof of the open percolation sufficiency direction, and not evidence that the three instruments share one admissibility predicate.

The successor question. For which recursive systems $x_{n+1} = \Pi(Tx_n)$ can the structural regime be certified from a finite description of (T, Π) and x_0 , without simulating the trajectory, and what is the exact boundary of that class? PRD-1 gives certificates for four families in a spectral frame; a successor should seek a family-independent criterion (for example in terms of invariant sets of $\Pi \circ T$) and a proof of necessity, not further confirmations. A separate, promising question is whether a rigorous map from ESC-I trajectories to STRUC-I ladders exists, and whether changes in projection regime then correspond to independently measured changes in ladder admissibility or percolation. That is a new research question, not a result of this manuscript.

12 Conclusion

What ESC-I has demonstrated. ESC-I develops the early UNNS complex-number programme into a testable mathematical framework. Real recursive representations and projection-aware structural classifications can be investigated with explicit rules, counterexamples and reproducible tests. Explicit conditions were established for two statements of the original manuscript; one preregistered rule was falsified, refined and re-confirmed; a projection-aware rule predicted the regime of $x_{n+1} = \Pi(Tx_n)$ without false certification in a stated domain, and an independent implementation found no contradiction on 2400 fresh cases. The spectrum of T alone does not determine the observed regime; the projection, its thresholds and the starting state do.

What ESC-I has not demonstrated. That the algebra of complex numbers emerges uniquely from UNNS generative admissibility without its defining structure being assumed; that its rules exceed classical dynamical-systems theory; or that its finite-horizon verdicts are asymptotic theorems.

Position within UNNS. Within the broader UNNS Substrate these findings give a specialised realisation of the distinction between structural generation, admissibility constraints and observable persistence, made mathematically explicit for complex-compatible real recurrences and nonlinear projections. The results neither replace the canonical admissibility and percolation instruments nor demonstrate a new universal law; they supply an additional, independently testable setting in which the consequences of recursive transformations can be classified and partially certified. The foundational question remains whether an independently specified UNNS generative grammar can select complex-compatible algebraic structure without a rotation operator assumed in advance; that requires a separate generative-selection theorem.

Closing the present experimental cycle is a statement about the chamber and not about the question. The broader question, whether admissibility of a recursive process can be characterised for more general systems without simulating trajectories, is the proper subject of a successor study.

A Proofs

Theorem A(i) $(UTU^{-1})^n(Ux_0) = U(T^n x_0)$ and U is a bijective bi-Lipschitz map, so boundedness, convergence, divergence, period and sub-type are preserved.

(ii) If $UT = TU$ and U, T are simultaneously triangularised, the eigenvalues of UT are the products $u_i t_i$ with $|u_i| = 1$, so $\rho(UT) = \rho(T)$. In 2×2 with $U = R_\theta$, $\theta \notin \{0, \pi\}$, the commutant of R_θ in $\mathbb{R}^{2 \times 2}$ is $\mathcal{C}$.

(iii) $\det R_\theta = 1$ gives $\det(R_\theta T) = \det T$. With $R_\theta = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$, $\text{tr}(R_\theta T) = (a + d) \cos \theta + (b - c) \sin \theta$ by direct multiplication.

(iv) The spectral radius of a real 2×2 matrix depends only on $(|\tau|, \Delta)$ with $\tau = \text{tr}$, $\Delta = \det$ and is nondecreasing in $|\tau|$. For $\Delta > 0$ it is constant on $|\tau| \leq 2\sqrt{\Delta}$ (complex pair, $\rho = \sqrt{\Delta}$) and strictly increasing beyond; for $\Delta \leq 0$ it is strictly increasing in $|\tau|$. As θ varies, $|\tau_\theta|$ sweeps $[0, m]$ with $m^2 = (a + d)^2 + (b - c)^2$. For $\Delta > 0$, θ -independence needs $m^2 \leq 4\Delta$; since $4\Delta - m^2 = -(a - d)^2 - (b + c)^2$ this forces $a = d$ and $b = -c$, i.e. $T \in \mathcal{C}$. For $\Delta \leq 0$ it needs $m = 0$, i.e. $a = -d$ and $b = c$, i.e. $T \in \bar{\mathcal{C}}$. The converse follows by direct computation. For the corollary, $\rho(UT) \leq \|UT\|_2 = \sigma_{\max}(T)$ for orthogonal U , with equality attainable when U aligns the top singular directions. $\square$

Theorem B(T4.1). By hypothesis $|u_{n+1}| \geq (|\lambda| - \eta)|u_n|$ and $|w_{n+1}| \leq (|\mu| + \eta)|u_n|$. Since $\eta \leq (|\lambda| - |\mu|)/2$ we have $|\mu| + \eta \leq |\lambda| - \eta$, so $|w_{n+1}| \leq (|\lambda| - \eta)|u_n| \leq |u_{n+1}|$; the ratio bound follows from $|u_{n+1}| \leq (|\lambda| + \eta)|u_n|$, and induction on n closes the argument. For the asymptotic statement let $r_n = |w_n|/|u_n| \leq 1$. Because $|z_n| = |u_n|$, $|w_{n+1}| \leq |\mu||w_n| + \eta|u_n|$ and $|u_{n+1}| \geq (|\lambda| - \eta)|u_n|$, so $r_{n+1} \leq a r_n + b$ with $a = |\mu|/(|\lambda| - \eta) \leq 2|\mu|/(|\lambda| + |\mu|) < 1$ and $b = \eta/(|\lambda| - \eta)$. Hence

$\limsup r_n \leq b/(1-a) = \eta/(|\lambda| - |\mu| - \eta)$. The bound is not claimed for finite n : for $r_0 = 1$ it can fail at $n = 0$. $\square$

(T4.2). $|u_{n+1}| \geq |\lambda||u_n| - h$, which exceeds $|u_n|$ iff $|u_n| > h/(|\lambda| - 1)$; once true it stays true because $|u_n|$ increases. Hence $|u_n| \rightarrow \infty$ and $|u_{n+1}|/|u_n| \rightarrow |\lambda|$. Writing $r_n = |w_n|/|u_n|$, $r_{n+1} \leq (|\mu|r_n + h/|u_n|)/(|\lambda| - h/|u_n|) \rightarrow 0$ because $|\mu| < |\lambda|$ and $|u_n| \rightarrow \infty$, so the direction converges to the dominant eigenline. Sharpness: the adversary $u \mapsto \text{sgn}(u) \max(|\lambda||u| - h, 0)$ is an admissible projection error of size h ; for $|u_0| \leq h/(|\lambda| - 1)$ it does not increase $|u|$. $\square$

B Counterexample inventory

Id	Data	Outcome (exact)
C-phase-1	$T = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, $U = \text{diag}(1, -1)$	$\rho(T) = 2$, $\rho(UT) = 0$, $\rho(UTU^{-1}) = 2$
C-phase-2	same T , $U = R_{\pi/2}$	$R_{\pi/2}T = \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix}$ nilpotent: ρ from 2 to 0
C-phase-3	$T = \text{diag}(2, \frac{1}{2})$, $R = \begin{pmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{pmatrix}$	RT : $\tau = \frac{3}{2}$, $\Delta = 1$, ρ from 2 to 1
C-proj-1	$T = \text{diag}(2, \frac{1}{2})$, C_1 zeroing, $x_0 = (\frac{2}{5}, 1)$	orbit $\equiv 0$ from step 1, though $\rho(T) = 2$
C-proj-2	same T , u -dead-zone $\theta = 1$, $x_0 = (\frac{2}{5}, 1)$	divergent $\rightarrow$ convergent along stable axis
C-proj-3	$F \equiv 0$, relative bound $ \lambda $	collapse; hypothesis $\eta < \lambda $ necessary
C-proj-4	$x_0 = (0, 1)$ on stable eigenline	convergent although $\rho = 2$
C-proj-5	$1.5R_{30^\circ}$, relative error $\eta = 0.3$	rate in $[1.2, 1.8]$; direction drift up to 40.3 rad in 200 steps
C-obs-1	$\rho = 1 + 10^{-7}$	echo reads bounded (needs $\sim 1.4 \times 10^8$ steps)
C-obs-2	$\tau = 1 + 10^{-12}$	echo reads period 6; exact class quasi-periodic
C-ctrl	$\lambda = -2, \mu = -1$, LINP($\frac{1}{2}$), $z_0 = (-4, -\frac{1}{2})$	II T orbit $\rightarrow$ 2-cycle; mapped to 0 in the frozen comparator (144 first-run contradictions)

C Reproducibility record

Item	Value
AR-1/2/3 rules hash	c32fa4fd2469fee5ab3654b91cb28843 e23463d04f5c56b7d63c7df14ec19b54
AR predictions hash	9e7cc9a7869faf247eb421bdd28ec73d 9cb9678dfd3e63d2f127e8c9b41b022e
AR-3.1 text hash	36d7bea38dd1b9217edc5a6654c7ca50 20fd93d019ed49dba91d56097965da41
AR-3.1 amended source hash	2cf5d68ae0b293fb2f06a37579e0dd21 6870ae3046efb0ec11cf6b9c713ff3f1
PRD-1 rules hash (seed 20261213)	3bf6d3666f28fa35ff017b35f4569af1 8cd420cd2a0faf985479d53ef7dab07d
PRD-1 predictions hash	fbe397db285357bc5328ce0d3a450e8e 35dfd3d53451b5d0907ea652c698a642
Seeds	AR 20261013; AR-3.1 20261113; PRD-1 20261213; PRD-1 fresh-seed amendment check 20261790; AR-3.1 independent 12345; PRD-1 independent 20261014
Main run	python <code>admissibility.py</code> (about 50 s); outputs in <code>output/</code>
Tests	python <code>test_esc_i.py</code> (11 tests, about 90 s)
Independent checks	python <code>independent_check_ar31.py</code> ; PRD1_VERIFY/RUN_VERIFY.bat
Figures	<code>synthesis/make_figures.py</code> (matplotlib; all six figures from ESC-I code and <code>output/prd1_report.json</code>)
Fresh-seed record	<code>synthesis/records/prd1_fresh_seed_20261790_report.json</code>

REPRODUCE.md explains what each level of reproduction establishes.

AR-3 sub-family	Cases	Agree	Disagree	Abstain
ADV_ABS (absolute adversary)	540	540	0	0
DEADZONE	540	432	0	108
ADV_REL (relative adversary)	1080	390	0	690
ROUND_ABS (nearest rounding)	540	417	0	123
REL_QUANT (k -bit quantiser)	540	207	92	241
Total	3240	1986	92	1162

Table 6: AR-3 frozen comparison by sub-family.

PRD-1 (frozen seed 20261213)	Cases	Predictions issued	Asymptotic	Finite-horizon	Abstain
R, dead-zone	1560	1560	1441	119	0
R, clipping	1080	1080	738	342	0
R, lattice	1170	951	951	0	219
R, state gate	387	387	29	358	0
C, dead-zone	1560	871	871	0	689
C, clipping	1080	0	0	0	1080
C, lattice	1170	907	907	0	263
C, state gate	387	0	0	0	387
C, linear control P (720) first run	720	720	576	—	144 contradicted
C, linear control P (720) amended	720	720	720	—	0 contradicted
C, linear control Q (720)	720	720	720	—	0 contradicted

Table 7: PRD-1 per-family counts as recorded in `prd1_report.json`.

D Full numerical summaries

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