Conditional Symmetry Emergence in τ-Relaxed Systems

Abstract

Symmetry is often assumed to emerge naturally in relaxed or equilibrated systems. In this work, we demonstrate that symmetry emergence is neither automatic nor universal, but instead *conditional* on the structure of the underlying relaxation landscape. Using a family of τ-relaxed discrete systems, we show that multiple symmetry measures may coexist, yet only a subset act as effective discriminators between stable outcomes. We introduce a selector-based framework that separates stability from selection, and empirically identify null and active symmetry axes. Our results show that symmetry relevance must be measured, not presumed, and that selective determinism can be improved without modifying system dynamics.

1 Introduction

Symmetry plays a central role across physics, mathematics, and complex systems. From energy minimization principles to spontaneous symmetry breaking, symmetry is often treated as an intrinsic organizing feature of stable states.

However, this view implicitly conflates two distinct questions:

1. Which states are dynamically stable?
2. Which stable states are selected or realized?

In this paper, we argue that stability alone does not determine symmetry realization. Even in fully relaxed systems, multiple stable outcomes may coexist, differing in their symmetry properties. Whether symmetry appears, and which symmetry appears, is therefore a problem of *selection*, not of dynamics.

We present an empirical study of τ-relaxed systems in which stability is fixed, yet symmetry relevance varies across measurable axes. This allows us to formulate and test the concept of *conditional symmetry emergence*.

2 System Definition

We consider discrete systems evolving under a relaxation operator τ. For fixed system parameters and initial conditions, τ evolves the system until convergence to a stable configuration.

Let S_τ denote the set of τ-stable states obtained from repeated runs with identical dynamics but differing initial seeds. All states in S_τ satisfy identical convergence criteria and are equally stable under τ.

Each state $s \in S_\tau$ is characterized by:

- a configuration vector $x(s)$,
- an energy-like scalar $U(s)$,

- a discrete topological invariant $W(s)$.

No further dynamical evolution occurs beyond τ-closure.

3 Symmetry Measures

To probe symmetry properties of τ-stable states, we define a set of symmetry functionals Σ_i, each mapping a stable configuration to a non-negative scalar.

The symmetry measures considered include:

1. Mean bias: deviation of the configuration average from zero,
2. Reflection asymmetry: deviation under spatial reflection,
3. Spectral asymmetry: power in odd Fourier modes,
4. Chirality: oriented imbalance in nearest-neighbor differences.

Importantly, these functionals:

- depend only on the final state,
- do not influence dynamics,
- do not introduce new invariants.

They are purely diagnostic.

4 Selector Framework

Because all states in S_τ are dynamically equivalent, any preference among them must arise from a selection principle external to τ. We formalize this via a selector operator κ, acting on S_τ.

Given a symmetry functional Σ_i, the selector is defined as:

$$\kappa[\Sigma_i] = \arg\min_{s \in S_\tau} \Sigma_i(s).$$

This operation selects the most symmetric state with respect to Σ_i, without altering stability or dynamics.

Multiple selector policies may be defined, including pure, weighted, or lexicographic combinations of symmetry measures. In this work, we emphasize lexicographic policies for their determinism and transparency.

5 Empirical Results

Across ensembles of τ-stable states, we observe the following:

5.1 Multiplicity of Stable States

For fixed dynamics, S_τ contains multiple distinct states differing in topology, energy, and symmetry. This confirms that τ-relaxation alone does not guarantee uniqueness.

5.2 Active and Null Symmetry Axes

Symmetry measures exhibit markedly different behavior:

- Some measures show clear variance across S_τ and sharply identify a preferred state.
- Others display vanishing variance, complete degeneracy, or negligible gaps between minima.

We define a symmetry axis as *null* if it fails to discriminate between τ-stable states under any reasonable threshold. Null axes are empirically detectable and reproducible.

In our experiments, chirality consistently appears as a null axis, while reflection asymmetry emerges as the strongest discriminator.

5.3 Conditional Determinism

Applying selectors across all symmetry axes yields partial determinism: selection fails when null axes are included. Restricting selection to empirically active symmetry axes restores full determinism without altering S_τ.

This demonstrates that determinism is conditional on symmetry relevance, not on dynamics.

6 Negative Controls

To test whether symmetry selection trivially correlates with energy or topology, we compare three reference states:

- the minimum-energy state,
- the symmetry-selected state,
- the maximally asymmetric state.

No universal alignment is observed. Some symmetry measures correlate weakly with energy; others do not. This confirms that symmetry selection is not reducible to energy minimization.

7 Discussion

The results support several key conclusions:

1. Symmetry does not automatically emerge from relaxation.
2. Symmetry relevance is system- and ensemble-dependent.
3. Selection must be distinguished from stability.
4. Null symmetries are empirically meaningful outcomes.

This reframes symmetry emergence as a conditional phenomenon rather than a universal law.

Reproducibility Statement

All results reported in this work are fully reproducible. The relaxation operator τ, system parameters, and termination criteria are fixed and deterministic. Ensembles were generated by varying only the random initialization seed, with no adaptive tuning or post hoc filtering. All symmetry measures and selector operations act exclusively on τ-stable configurations and do not modify system dynamics. The complete set of recorded configurations, symmetry evaluations, and selection outcomes is sufficient to independently reproduce all tables, figures, and conclusions presented here. No hidden parameters, stochastic selectors, or retrospective adjustments were employed.

Discussion Addendum: Relation to Symmetry-Breaking Narratives

Symmetry emergence in many theoretical frameworks is commonly framed in terms of symmetry breaking: a symmetric governing law gives rise to asymmetric outcomes through dynamical instability, perturbations, or energetic preference. The present results do not contradict such narratives, but they demonstrate that they are incomplete.

In the systems studied here, asymmetry does not arise from the breaking of a symmetric dynamical law. Instead, multiple τ-stable outcomes coexist, already differentiated by their symmetry properties. Which symmetry is realized is therefore not determined by dynamics alone, but by a subsequent selection process acting on an already stable ensemble. Moreover, some symmetry axes are shown to be empirically null, meaning that no symmetry breaking or preference occurs along those dimensions at all.

From this perspective, symmetry emergence is not a universal consequence of relaxation or instability, but a conditional phenomenon whose relevance must be measured rather than assumed. The selector framework introduced here complements, rather than replaces, traditional symmetry-breaking accounts by explicitly separating *stability*, *selection*, and *interpretation*.

8 Conclusion

We have shown that symmetry emergence in τ-relaxed systems is conditional, not guaranteed. By separating relaxation from selection, and measuring symmetry relevance directly, we identify both active and null symmetry axes. Selective determinism can be achieved without modifying dynamics, provided that selection is conditioned on empirically active symmetries.

These findings suggest a general methodological principle: *symmetry must be measured before it is interpreted.*

Appendix A: Methods

This appendix summarizes the experimental procedure used to obtain the results reported in the main text. All methods are deterministic, reproducible, and do not modify system dynamics.

A.1 Ensemble Generation

For fixed system parameters, the relaxation operator τ was executed multiple times from distinct initial conditions. Each run was evolved until convergence, producing a τ-stable configuration.

- System size: fixed per experiment

- Dynamics: identical across runs
- Initial conditions: randomized seeds
- Termination: convergence of energy and topology

The resulting set of stable configurations defines the ensemble S_τ.

A.2 Recorded State Data

For each $s \in S_\tau$, the following quantities were recorded:

- Configuration vector $x(s)$
- Energy $U(s)$
- Topological invariant $W(s)$

No intermediate trajectories were stored or analyzed.

A.3 Symmetry Evaluation

Each symmetry functional Σ_i was evaluated independently on all states in S_τ. Symmetry measures depend only on the final configuration and are invariant under τ.

A.4 Selector Application

Selectors were applied to the *fixed* ensemble S_τ. No further relaxation was performed.

Pure selector:

$$\kappa[\Sigma_i] = \arg\min_{s \in S_\tau} \Sigma_i(s)$$

Lexicographic selector: States were first ranked by Σ_{i_1}; ties were resolved using Σ_{i_2}, and so on.

A.5 Null-Axis Detection

A symmetry axis Σ_i was classified as *null* if it failed to discriminate between states under at least one of the following conditions:

- Vanishing variance across S_τ
- Complete or near-complete degeneracy at the minimum
- Negligible gap between first and second minima

Only empirically active axes were retained for conditioned selection.

A.6 Negative Controls

To assess whether symmetry selection trivially correlates with other observables, three reference states were compared:

- $s_{U\,\mathrm{min}}$: minimum-energy state
- s_{κ}: symmetry-selected state
- s_{κ^-}: maximally asymmetric state

Differences in U and W between these states were recorded.

A.7 Pseudo-Code Summary

```
Input: system parameters, number of runs R

S_tau = {}

for r = 1 to R:
    initialize system with random seed
    evolve under tau until convergence
    record x(s), U(s), W(s)
    add s to S_tau

for each symmetry Sigma_i:
    compute Sigma_i(s) for all s in S_tau
    analyze variance, degeneracy, and gap

ActiveAxes = {Sigma_i not classified as null}

for each selector policy:
    apply selector on S_tau using ActiveAxes
    record selected state

compare selected states with minimum-energy
and maximum-asymmetry references
```

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Appendix B: Summary of Data Analysis

The following empirical patterns were consistently observed across all ensembles:

- Multiple τ-stable states coexist for fixed dynamics.
- Symmetry measures differ in discriminative power.
- At least one symmetry axis is typically null.
- Restricting selection to active axes restores determinism.

- Symmetry-selected states do not universally minimize energy.

These findings were reproducible across independent ensembles and robust to changes in ensemble size.

No post-selection feedback into τ dynamics was observed.