Beyond Integer Depth: Multi-Clock Recurrence, Prospective Falsification, and a Fractional-Primary Boundary

A Companion Study to the Integer-Depth Temporal-Recurrence Grammar

UNNS Research Collective
UNNS Substrate Research Program

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Abstract

A companion manuscript in this program (TIME-CRYSTAL-I, hereafter Manuscript I [7]) froze an integer-depth recurrence grammar, validated it prospectively on two digital-quantum-computer/numerical discrete-time-crystal (DTC) candidates, and encountered a domain boundary at a third candidate, C003, a robust experimental two-clock discrete time quasi-crystal (DTQC) [6]. Manuscript I deliberately did not develop a multi-clock successor: it named vector-depth, toroidal, and incommensurate closure coordinates as motivated-but-undeveloped future work and required that any such grammar be independently specified and tested rather than retrofitted to C003. This manuscript reports that program, carried out on a separate development corpus (Huang [8], Luo [9], and, after a locked transfer test converted it into retrospective evidence, Zhu [10]) that deliberately excludes C003 throughout. Four representations were baked off. Joint-phase-conditioned recurrence (JPR), evaluated relative to a source-defined drive torus, produced a source-torus anchor coordinate (P_{src}) and a non-gating chart coordinate (J_{frac}). A separate fractional-cover decomposition — developed only after a locked transfer test on protected holdout data (Zhu) exposed a cover-depth-selection limitation in an earlier drive-torus regression diagnostic, and after the decomposition itself survived a pre-freeze specificity challenge on Malz–Smith topological-qubit data [11] — produced the fractional mixed-clock gain (M_{frac}), which alone separated four development-stage DTQC-positive records from sixteen controls at AUC = 1.000. The resulting grammar, `MC_GRAMMAR_v001`, was frozen and then subjected to a genuinely prospective, blind-lock-reveal test against an external theoretical-model transfer (Marripour–Abouie [12]): the predeclared candidate record was expected to be TEMPORAL_CORE_SUPPORTED and instead returned MIXED_ORGANIZATION_WEAK with $M_{frac} \approx -0.330$, the opposite sign from every development-stage positive. This is reported as a prospective failure, not repaired after the fact. Only afterward was C003 reopened under the frozen grammar. It remains formally outside the grammar's supported class (P_{src} fails at $p = 1.000$), yet its fractional multi-clock coordinates are the strongest seen anywhere in this study (JPR-derived $J_{frac} \approx 0.911$; FC-derived $M_{frac} \approx 0.596$, stable under all six frozen robustness transforms) and sharply separate it from its matched breakdown control ($J_{frac} \approx -0.594$, $M_{frac} \approx -0.004$). The resulting pattern — weak parent-torus anchoring together with strong fractional organization — is close to the structural inverse of what `MC_GRAMMAR_v001` was built to expect, and is not observed in any development-corpus DTQC-positive record. We do not claim that C003 is admitted by the frozen grammar, that the grammar is universal, or that a second physical species of multi-clock order has been established. We report a narrower, better-earned result: an independently developed and frozen multi-clock grammar failed its first prospective test and, when applied unchanged to the historically quarantined boundary case, exposed a second

structural boundary inside multi-clock temporal organization itself — motivating a parent-anchored/fractional-primary distinction as a falsifiable research question for a future grammar version, rather than as a conclusion of this one.

Contents

1 Introduction — The Boundary Left by the Integer Grammar

Manuscript I asked a structural-classification question about discrete time-crystalline recurrence [1, 2, 3, 4, 5] and answered it with a frozen integer-depth recurrence-closure grammar, $\mathrm{closure}(x, y) = \mathrm{similarity}(x, y) \times \mathrm{support}(x, y)$, evaluated at a data-selected fundamental depth q_0 and gated jointly on closure $C(q_0)$, family contrast F, and shuffle-null significance p. Under that frozen grammar,

$$X_t \to X_{t+q},$$

with q an integer separation in an ordered temporal sequence, two prospective DTC-related candidates (c001, c002) independently entered the supported class at $q_0 = 4$ and $q_0 = 2$ respectively, while their controls did not. A third candidate, c003 — a robust experimental $\mathbb{Z}_2$ discrete time quasi-crystal with two incommensurate drive clocks — did not: both the candidate ($q_0 = 6$, $C = 0.1988$, $F = 0.0934$, $p = 0.9602$) and its matched breakdown-regime control returned NO_TEMPORAL_ORDER under the identical frozen gate.

Manuscript I was explicit about what that result did and did not mean. It did *not* conclude that c003 lacks temporal order; it concluded that c003 lies outside the supported domain of the frozen integer-depth grammar, and it introduced — without developing — candidate coordinates for a successor: vector recurrence depth (q_1, q_2), toroidal phase closure on $\mathbb{T}^2$, and irrational/incommensurate recurrence ratios. Its own words were precise about the obligation this created: these are hypotheses motivated by c003, not results of that manuscript, and any such branch requires its own unseen validation data rather than retrofitting into TIME-CRYSTAL-I v1.1.0. Its conclusion likewise called for "a separate prospective treatment of multi-clock quasi-periodic organization in future work."

This manuscript is that separate treatment. It is deliberately not an extension, appendix, or revised edition of Manuscript I; it is developed, frozen, and prospectively tested on its own corpus, under its own firewall, and only afterward brought back into contact with c003. We state at the outset what this manuscript is *not*: it is not a "TIME-CRYSTAL-II successfully detects DTQCs" report. That is not the result the frozen grammar produced. The result is more structured than a single positive or negative verdict, and section 2 states the methodological chronology that makes it possible to trust the distinction between what was development evidence, what was frozen before being tested, and what was genuinely unseen.

What follows. An independently developed and frozen multi-clock structural grammar was prospectively challenged, its prospective failure was preserved rather than repaired, and the unchanged grammar was then used to interrogate the historically quarantined c003 boundary case. The outcome is richer than simple admission of the historical boundary case: the first frozen multi-clock grammar itself encounters a new structural boundary, one that the grammar's own architecture — an implicit assumption that parent-torus anchoring precedes fractional organization — was not designed to separate from genuine non-order.

2 Methodological Promise and Research Firewall

2.1 The chronological firewall

Every claim below depends on the reader being able to verify *when* c003 was permitted to touch this branch's development. The firewall enforced throughout, and audited in §15, is:

TIME-CRYSTAL-I boundary → c003 quarantine → independent multi-clock development corpus
→ representation bake-off → drive-torus regression diagnostic (v002) → **locked Zhu v002 transfer test**
→ fractional-cover v003 → pre-freeze Malz–Smith specificity challenge
→ four-representation study → nulls + robustness + control discrimination
→ compact grammar selection → grammar freeze
→ P001 prospective blind test → reveal → c003 final historical test.

Figure 1 renders this chain explicitly. c003 sits at the far end, after every development, selection, and freezing decision — not, as a retrofit would require, near the beginning. The locked Zhu transfer test (bold above) is highlighted deliberately: it is the pivot at which the v002 drive-torus diagnostic was judged insufficient and retired in favor of the fractional-cover v003 representation — its provenance is introduced in §3.1 and its technical detail given in §5.1 — not a step that can be silently collapsed into either its neighbor.

2.2 Explicit prohibitions

c003 was not used, at any point prior to §10, for: representation selection; coordinate design; threshold setting; null-model design; robustness design; compact grammar selection; or grammar freeze. The prospective P001 test (§9) was likewise not used to modify the frozen grammar after reveal — a result inconvenient to the grammar's own hoped-for generality was preserved, not patched.

2.3 Provenance

The frozen grammar (`MC_GRAMMAR_v001`, §8) carries SHA-256 checksum

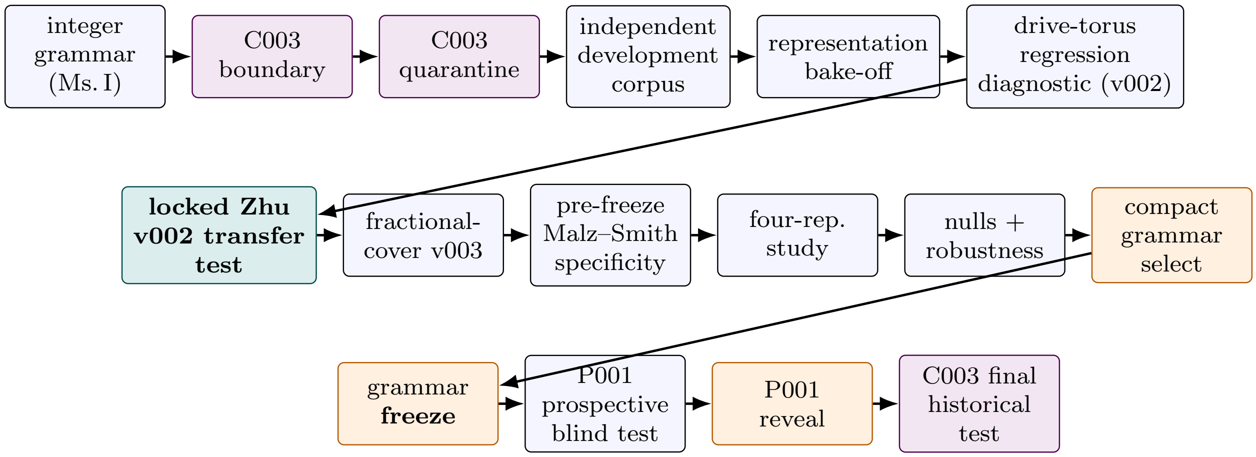

Figure 1: The chronological firewall enforced throughout this branch. Violet boxes mark every point at which C003 is permitted to touch the analysis: the historical boundary it originally exposed (in Manuscript I), and its final, single, post-freeze evaluation. The teal box marks the locked Zhu transfer test — a genuine holdout evaluation of the v002 drive-torus regression diagnostic, executed and resolved (`PARTIAL_TRANSFER_WITH_COVER_DEPTH_FAILURE`) before fractional-cover v003 was designed, and distinct from the fractional-cover development it motivated. Everything between the two violet nodes — the full development, selection, and freezing chain — used a corpus from which C003 was excluded by protocol.

`8ae9b0d91dba41332dac58392b84a222f6c63b6323b8ddb616ce39e4bba556d6`

recorded in `FROZEN_GRAMMAR.json` before any prospective candidate existed to the chamber. Any post-freeze methodological change — a different threshold, a different null, a different robustness transform, a relaxed ratio domain — would require declaring a new grammar version (`MC_GRAMMAR_v002` or later), not patching v001 in place. This rule is stated here because it is exercised, not merely asserted: §9 reports a case in which it was followed under real pressure to do otherwise.

3 Corpus Provenance and Claim Ledger

3.1 Corpus roles

Table 1 summarizes the role and status of every source touching this branch. Two entries require chronological unpacking because their role changed mid-branch, and conflating their earlier and later roles would overstate what they can support.

Zhu. ZHU entered as a protected holdout under the research firewall ("ZHU_2026 is reserved as a holdout and must not be analyzed during development"). A locked test, `ZHU_VALIDATION_v001`, was pre-registered to ask a narrow external-transfer question of the *v002* drive-torus method (§5): does it generalize to an independent experimental DTQC source it had never seen? The result was `PARTIAL_TRANSFER_WITH_COVER_DEPTH_FAILURE` — the underlying two-clock organization genuinely transferred, but v002's own cover-depth selector failed to pick it out correctly. Because this outcome was mixed and method-revising rather than a clean pass, the two released records (Fig. 1 C1, C3) were reclassified `DTQC_POSITIVE` and folded into the development corpus. Most consequentially, Zhu C1's fractional mixed gain ($M_{frac} = 0.138611$) became the empirical lower bound that sets the frozen grammar's raw M_{frac} gate (§8). A record that helps define a threshold cannot also independently validate that threshold: Zhu's role in this branch is retrospective and formative, not confirmatory.

Malz–Smith. MALZ–SMITH topological-qubit tomography data were used in a locked, pre-freeze specificity challenge of the fractional-cover v003 construction (§7) — a genuine, if narrow, held-out test executed *before* the four-representation study and grammar development that followed it. The challenge passed strongly. Its 12 records were then incorporated as development controls in the later representation study and grammar-development stages (§§4–7). It is therefore not post-freeze validation of `MC_GRAMMAR_v001`; it is a pre-freeze specificity test whose records were subsequently repurposed as development evidence.

c003. The historical quarantine case. Used only after freeze, after the P001 prospective analysis, and after reveal (§10).

Table 1: Corpus provenance and role. "Eligible as validation" asks whether a source's evidentiary status *permits* it to test the frozen grammar prospectively or independently — as opposed to having contributed to building it. Eligibility is a methodological property, not an outcome: it does not imply the test passed. P001, for example, is eligible and genuinely prospective, but its candidate hypothesis failed (§9).

Source	Physical system	Records	Final role	Eligible as validation
Huang [8]	continuous-time spin dynamics	3	source-free cross-chart controls (LC/QP/chaos)	no
Luo [9]	Rydberg-chain numerical DTQC	10	core development corpus (DTQC positive, LOW/HIGH controls)	no
Zhu [10]	experimental DTQC	2 released	holdout → locked transfer test → reclassified development evidence; sets the M_{frac} gate lower bound	**no**
Malz–Smith [11]	experimental topological Floquet qubit	12	pre-freeze specificity challenge (pass) → development controls	**no** (pre-freeze only)
Marripour–Abouie [12] (P001)	external theoretical-model transfer	2 (A/B)	genuinely prospective candidate/control, post-freeze	**yes**
c003 [6]	experimental $\mathbb{Z}_2$ DTQC (historical)	1 + matched control	historical quarantine case, evaluated once post-freeze	**no** — historical post-freeze retest only

3.2 Claim ledger

Demonstrated.

- [development-stage] M_{frac} is the strongest development-stage discriminator among the tested temporal coordinates (AUC = 1.000 against all four development control families).
- [development-stage] The fractional-cover v003 representation survives a pre-freeze specificity challenge against generic two-clock quasiperiodicity (Malz–Smith [11]).
- [historical retest] C003 exhibits a strong, null-resistant fractional signature absent from its matched breakdown control.
- [development-stage] Temporal multi-clock organization and collective coupling are empirically distinct axes in the Luo regimes.

Explicitly not claimed.

- [not claimed] `MC_GRAMMAR_v001` is universal.
- [not claimed] The grammar prospectively validated a general DTQC detector.
- [not claimed] C003 is admitted by the frozen grammar.
- [not claimed] Parent-anchored and fractional-primary organization are already established as universal physical classes.
- [not claimed] A universal numerical collective gate (C_{coll}) exists.

Empirically contradicted within the tested corpus.

- [contradicted in-corpus] Mere two-clock quasiperiodicity is sufficient for the fractional signature (refuted by Malz–Smith, §7).
- [contradicted in-corpus] Total fractional organization alone identifies the DTQC regime (refuted: `FC_frac_total_gain` fails DTQC-vs-Luo-HIGH discrimination, AUC = 0.500; §7).
- [contradicted in-corpus] Temporal organization is equivalent to collective coupling (refuted by the Luo regime ordering; §12).

4 Representation Bake-Off

Four candidate representations were compared on the same eligible development records before any one of them was allowed to define a decision coordinate: vector-depth recurrence, joint-phase-conditioned recurrence (JPR), a frequency lattice, and fractional-cover decomposition. None collapses to a single structural quantity, and each was tried, in roughly this order, before the eventual grammar's two gating coordinates (P_{src} from JPR, M_{frac} from fractional cover) emerged.

Vector-depth recurrence. Useful as a recurrence chart, but too general: it also lights up for ordinary integer recurrence and generic smooth temporal organization, and does not by itself separate DTQC from either same-system control (role-level family contrast 0.0611 for DTQC versus 0.0144–0.2699 across controls; Table 2).

Frequency lattice. Detects genuine combination-frequency organization, but is too permissive: generic two-clock quasiperiodicity can produce strong lattice alignment. In the first-pass bake-off the high-frequency decoupled control's lattice gain (16.43) was substantial despite decoupling, though it did not exceed the DTQC-positive record's first-pass value (23.66); the genuine inversion appears only later, at the role-median stage of the four-representation study (Table 2), where the high-frequency control's peak contrast (5.4555) *does* exceed the DTQC-positive role median (2.8956).

Joint-phase-conditioned recurrence. Introduces a genuinely new chart coordinate — are similar physical states preferentially encountered near the same joint source phase? — evaluated relative to a source-defined drive torus $(\theta_L, \theta_R) = (2\pi x, 2\pi\varphi x)$ rather than an observable-derived one (§5).

Fractional-cover decomposition. Provides the strongest development-stage discrimination through mixed fractional clock organization (§6).

Table 2: Role-level medians across the four representations (`REP_STUDY_v002/GROUP_SUMMARY.json`), with pairwise AUC against Luo LOW/HIGH and Malz–Smith topological/trivial (`GRAMMAR_DEV_v001`). M_{frac} (FC mixed gain) is the only column in which DTQC_POSITIVE sits strictly above every control role, and the only coordinate with AUC = 1.000 against all four control families.

Role	VD contrast	P_{src} (raw)	J_{frac}	FL contrast	FC parent R^2	M_{frac}	FC total
DTQC_POSITIVE	0.0611	0.3088	0.3616	2.8956	0.2674	**0.2244**	0.6156
Luo LOW	0.0022	0.1427	0.3981	1.0812	0.0020	0.0528	0.1300
Luo HIGH	0.0144	0.0699	0.6025	5.4555	0.0358	0.1005	0.6986
Malz–Smith topo.	0.2699	0.6309	−0.2552	1.3103	0.8798	−0.0092	−0.0163
Malz–Smith trivial	0.0816	0.1575	−0.0771	0.6620	0.1883	−0.0549	−0.0898

Pairwise AUC (min. over the four control families listed): J_{frac} same-system 0.500, generic-QP 1.000; M_{frac} same-system **1.000**, generic-QP **1.000**; FC total same-system 0.500, generic-QP 1.000; P_{src} raw same-system 0.750, generic-QP 0.125.

5 Source-Torus Geometry and Joint-Phase Recurrence

Two representations, not one — a note on exposition order. This section presents the *v002 source-defined drive-torus regression diagnostic*, the branch's earliest attempt to formalize source-anchored recurrence. It is a genuine historical predecessor of, but is representationally and numerically distinct from, joint-phase-conditioned recurrence (JPR) — the representation whose role-level outputs (P_{src}, J_{frac}) already appear in Table 2 (§4). The two must not be read as the same quantity under different names: the v002 diagnostic's cross-validated regression R^2 for the DTQC regime (0.9409, Table 3 below) is not the same number as JPR's role-median parent gain ($P_{src} = 0.3088$) or cover advantage ($J_{frac} = 0.3616$) in Table 2. The v002 diagnostic was retired — not extended — after the locked Zhu transfer test below exposed a specific failure mode in its cover-depth selector; JPR and fractional-cover v003 were developed afterward as its two independent successors (Figure 1).

For externally specified clocks, source phase is defined directly from drive metadata rather than

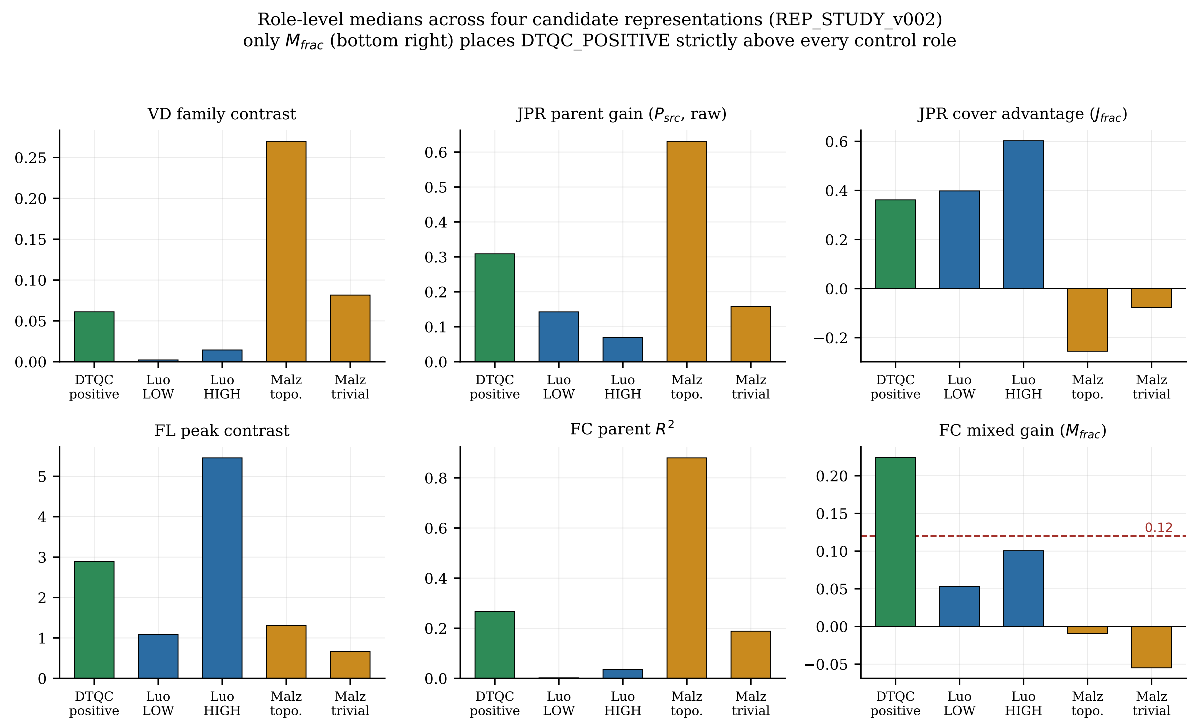

Figure 2: Role-level medians across the four candidate representations, one panel per coordinate. Only the fractional mixed-clock gain M_{frac} (bottom right, dashed line at the eventual frozen raw gate 0.12) places DTQC_POSITIVE strictly above every control role. Vector-depth contrast, JPR parent gain, and frequency-lattice contrast are each defeated by at least one control; JPR cover advantage separates DTQC from the two Malz–Smith roles but not from the two Luo same-system controls.

reconstructed from the observable itself:

$$\phi_1(t) = \omega_1 t \pmod{2\pi}, \qquad \phi_2(t) = \omega_2 t \pmod{2\pi}.$$

The trajectory is thus studied relative to a *source torus* rather than a single integer recurrence ladder. In practice the v002 diagnostic used a normalized source-defined torus $(\theta_L, \theta_R) = (2\pi x,\, 2\pi\varphi x)$, contrasted directly against an observable-derived torus, and fit by cross-validated regression. On six Luo [9] Fig. 3 entanglement trajectories (Table 3), the source-defined torus's full cross-validated regression R^2 ranks the intermediate DTQC regime (0.9409) strictly above both same-system controls (low-frequency aperiodic 0.1213; high-frequency decoupled 0.6976) — the first single coordinate in the branch to do so, at $N = 12$ and $N = 32$ alike (not a finite-size artifact), and stable under $\times 5/\times 10$ downsampling, first-75% truncation, and 2% RMS noise.

Table 3: The v002 source-defined drive-torus regression diagnostic versus an observable-derived torus, six Luo Fig. 3 entanglement trajectories, $N = 32$. These are cross-validated torus-regression quantities (full/axis-only R^2, regression mixed-term gain) — an earlier, numerically distinct diagnostic from the JPR representation reported in Table 2, not a restatement of it (see note above).

Regime	best depth	full R^2 (cv)	axis-only R^2	regr. mixed gain	mean S
Low-frequency aperiodic	2	0.1213	0.0697	0.0515	0.5853
Intermediate DTQC	2	**0.9409**	0.7167	**0.2242**	0.2425
High-frequency decoupled	2	0.6976	0.5972	0.1004	0.0719

The regression's *mixed-term gain* — the improvement in the v002 torus-regression fit from adding cross-clock terms beyond a per-axis fit — is the first *explicitly cross-clock interaction* coordinate to show the DTQC > HIGH > LOW ordering (the full-model R^2 in the same table already ranks the regimes this way, so the mixed-term gain is not the first coordinate to separate DTQC from both controls outright, but the first to do so via a term that is specifically cross-clock rather than a joint per-axis fit): $\approx 2.23\times$ the high-frequency control's mixed gain and $\approx 4.35\times$ the low-frequency control's. But the physical coupling-strength proxy (mean entanglement entropy S) runs in the *opposite* order: low (0.5853) > DTQC (0.2425) > high (0.0719). This is the first appearance, at the diagnostic stage, of the temporal/collective distinction developed fully in §12. Specificity against generic (non-DTQC) two-clock quasiperiodicity was not yet established here; that required the Malz–Smith challenge (§7).

5.1 The locked Zhu transfer test: why v002 was retired

Before any further development, a locked test (`ZHU_VALIDATION_v001`) asked whether the v002 diagnostic above generalizes to an independent experimental DTQC source it had never seen (§3.1). The result, `PARTIAL_TRANSFER_WITH_COVER_DEPTH_FAILURE`, showed that the underlying source-anchored organization genuinely transfers, but v002's own cover-depth selector ($\texttt{argmax}_d$ $\texttt{full\_r2\_cv}(d)$) does not pick it out correctly on external data. This mixed outcome — real signal, wrong selector — is what motivated fractional-cover v003 (§6) as a successor, and it is why this manuscript treats the v002 numbers above as a closed, historical diagnostic rather than as the source of the grammar's eventual gating coordinates.

5.2 The JPR cover-advantage coordinate

$$J_{frac} = \max_{d\in\{2,3,4\}} \mathrm{JPR}(d) - \mathrm{JPR}(1).$$

Interpretation: $J_{frac} > 0$ indicates a fractional cover organizes recurrence better than the integer parent torus; $J_{frac} < 0$ indicates the parent torus remains structurally dominant.

Critical methodological point. J_{frac} is retained throughout this manuscript as a *chart coordinate*, never a hard admission gate (Table 6). It separates DTQC from generic two-clock quasiperiodicity (AUC 1.000 against both Malz–Smith roles) but does not separate Luo DTQC from its own LOW/HIGH controls strongly enough (AUC 0.500 against both), and it is more sensitive than M_{frac} under some robustness transforms. Its role in this grammar is diagnostic, not decisive — a distinction that becomes central again when C003 is reopened in §10, since it is precisely J_{frac} (not M_{frac}) that carries C003's most extreme value.

6 Fractional Mixed-Clock Organization

At fractional cover depth d, two quantities are compared: the fractional-axis basis (separate per-clock fractional terms) and the fractional-full basis (axis terms plus mixed cross-clock combinations). The selected depth is

$$d^* = \arg\max_{d \in \{2,3,4\}} \Delta R^2_{\text{total}}(d),$$

and the primary decision coordinate is

$$M_{frac} = R^2_{\text{full}}(d^*) - R^2_{\text{axis}}(d^*).$$

M_{frac} measures the incremental organization contributed by *mixed fractional combinations of both clocks*, beyond separate fractional axis terms (Table 4).

Table 4: Fractional-cover decomposition. **Left:** $N = 32$ Luo development records. **Right:** the two Zhu retrospective records that motivated the v003 fractional-cover construction and highlighted the mixed term later selected as M_{frac} (Zhu was used to design v003 and is not independent validation of it).

Regime	Parent R^2	Frac. axis	Frac. mixed	Frac. total
LOW	0.002072	0.076870	0.051774	0.128645
DTQC	0.012515	0.717709	**0.224366**	0.942075
HIGH	0.035786	0.598050	0.100526	0.698576

Record	Parent R^2	Frac. axis	Frac. mixed
C1	0.600458	−0.043640	**0.138611**
C3	0.522323	−0.011398	**0.300533**

Zhu's fractional-mixed-gain values are strongly positive despite near-zero or negative fractional axis gain — the first hint that the *mixed* term, not the axis or total term, is where the specific signal lives. `FC_frac_total_gain`, despite strong raw values in the Luo development table, was dropped as a decision coordinate (§7) because it is strongly redundant with M_{frac} ($\rho \approx 0.949$) and fails DTQC-vs-Luo-HIGH discrimination outright (AUC = 0.500). `FC_parent_full_r2` was dropped for the same redundancy reason relative to P_{src} ($\rho \approx 0.887$), adding no unique compact-grammar role.

Development-stage discrimination: M_{frac} achieves AUC = 1.000 against all four development-control families (Luo LOW, Luo HIGH, Malz–Smith topological, Malz–Smith trivial) — the only coordinate tested anywhere in this branch with that property (Table 2).

7 Specificity, Nulls, and Robustness

7.1 Pre-freeze specificity challenge: Malz–Smith

Before the four-representation study or any grammar development, a locked, pre-analysis specificity challenge asked a falsifying question of fractional-cover v003: does merely having two incommensurate clocks — with no DTQC order at all — automatically produce a positive fractional-cover signature? If yes, the branch's central intuition collapses. Twelve experimental topological-Floquet-qubit tomography runs [11] (36 scalar $X/Y/Z$ component records, spanning $0.6 \leq M \leq 3.4$, both topological $C = -1$ and trivial $C = 0$ regimes) were tested. The v003 fractional-cover representation's own parent-torus fit is substantial in most components (median FC parent R^2: 0.86–0.90 across all three topological components; 0.31–0.40 for trivial X/Y, but only ≈ -0.14 for trivial Z) — this is the fractional-cover decomposition's own parent term, *not* the later JPR-based P_{src} null gate (§8), and it should not be read as "a strong P_{src} signal"; the two metrics are computed by different representations and the trivial regime's anchoring is not uniform across components. What matters for specificity is not this parent-level fit but the fractional gain reported below, which is negative throughout: median fractional total gain was *negative in all six regime×component cells* (Table 5), and robustness checks (downsample $\times 5/\times 10$, first-75% truncation, 2% noise) kept the regime medians negative throughout; only 1/18 topological and 2/18 trivial scalar records showed any positive `frac_total_gain` at all, consistent with noise rather than signal.

Table 5: Malz–Smith pre-freeze specificity challenge: median `frac_total_gain` by regime and component. Every cell is negative — the opposite sign from Luo DTQC ($\approx +0.94$) and Zhu (+0.09, +0.29).

Component	Topological ($C = -1$)	Trivial ($C = 0$)
X	−0.004914	−0.026201
Y	−0.007267	−0.029130
Z	−0.008673	−0.161759

This is a genuine specificity result — **[development-stage]** v003 is not fooled by mere two-clock incommensurability — but its own report was explicit about its limit: it demonstrates that v003 does not fire on generic two-clock quasiperiodicity, not that it is a validated DTQC detector in general. It was a locked, pre-freeze challenge, executed and interpreted before the four-representation study, not an independent validation run against the finished grammar; its 12 records were subsequently incorporated into the development corpus as controls (§3.1), anchoring the `QP_NON_DTQC_TOPO` and `QP_NON_DTQC_TRIVIAL` role rows throughout §4–§8.

7.2 Null models

`PHASE_LABEL_PERMUTE` (for P_{src}). The physical trajectory is preserved while source-torus labels are reassigned, directly testing the physical-state/source-torus relationship. Frozen ensemble: 99 surrogates; upper-tail Monte Carlo probability.

`FOURIER_PHASE` (for M_{frac}). Preserves spectral power while destroying the original temporal phase realization. Frozen ensemble: 49 surrogates; ridge $\alpha = 10^{-9}$, 5-fold cross-validation; upper-tail Monte Carlo probability.

The same `PHASE_LABEL_PERMUTE` null was also applied, diagnostically, to the non-gating J_{frac} coordinate — this is a supplementary application of the null, *not* the frozen P_{src} gate itself, since

J_{frac} is never gated (§5): median DTQC-positive percentile under `PHASE_LABEL_PERMUTE` for J_{frac} is 0.803 (Luo-low 0.990, Luo-high 1.000, Malz-topo 0.000, Malz-trivial 0.045). P_{src}'s own per-record null outcomes (the actual frozen gate) are reported directly, per record, in Tables 7 and 8 rather than as an aggregate role-level percentile. Separately, median DTQC percentile for M_{frac} under `TIME_PERMUTE/BLOCK_SHUFFLE/FOURIER_PHASE`: 1.000/1.000/0.990.

7.3 Candidate gate derivation

Using the exact development values,

$$\min\bigl(M_{frac}^{\mathrm{DTQC}}\bigr) \approx 0.138611 \quad \text{(Zhu C1)}, \qquad \max\bigl(M_{frac}^{\mathrm{control}}\bigr) \approx 0.100526 \quad \text{(Luo HIGH)},$$

with midpoint 0.119568 . . ., giving the frozen candidate boundary

$$M_{frac} \geq 0.12.$$

This uses the *maximum control development value*, not a control median — a deliberately conservative gate. Development replay at selection: 4/4 DTQC-positive core satisfied; 0/16 control core satisfied.

7.4 Robustness suite

Six frozen transforms: temporal-origin shift, clock exchange, affine amplitude transformation, $\times 5$ downsampling, first 75% of trajectory, and 2% RMS noise. Across a 30-row perturbation sweep, all 4/4 DTQC-positive records remain above the raw gate under each of the six transforms (median M_{frac} range 0.2243–0.2291 across variants), and both control roles hold at 0/2 or 0/12 across all variants — with a single documented exception: the Luo high-frequency decoupled control can cross the *raw* $M_{frac} = 0.12$ threshold under the 75%-prefix transform (median $M_{frac} \to 0.1480$). We state precisely why this is not a frozen-grammar false positive: the robustness transforms are an additional stability requirement on the base classification, not alternative classification inputs in their own right, and the HIGH record already fails the base $M_{frac} \geq 0.12$ condition before this transform is applied — it is preserved here as a documented boundary condition of the raw gate considered in isolation, not smoothed over.

8 Compact Grammar Selection and Freeze

8.1 Selected minimal grammar

$$D_{2clk} \to \bigl[\,P_{src}\,;\, J_{frac} \mid M_{frac}\,\bigr] \;+\; C_{coll}.$$

8.2 Domain clause

Requires exactly two externally specified incommensurate clocks; a genuine time-domain observable; source ratio taken from external metadata (not fitted); no response-derived clock; no synthetic inverse reconstruction. Supported ratio domain at v001: **golden-ratio branch only** ($\varphi = 1.618033988749895$); non-golden-ratio robustness is explicitly untested at this stage.

Table 6: Compact grammar freeze: every candidate coordinate considered, and why each was kept or dropped. "Same-system min AUC" is worst-case discrimination against the physically closest control (Luo LOW/HIGH); "generic-QP min AUC" is worst-case against the physically unrelated Malz–Smith controls.

Coordinate	Symbol	Same-sys. min AUC	Generic-QP min AUC	Decision
JPR parent gain	P_{src}	0.750	0.125	**select** — null-anchor gate
JPR cover advantage	J_{frac}	0.500	1.000	**select** — chart coordinate, non-gating
FC parent full R^2	P_{FC}	0.500	0.083	drop — redundant with P_{src} ($\rho \approx 0.887$)
FC frac. mixed gain	M_{frac}	**1.000**	**1.000**	**select** — primary temporal gate
FC frac. total gain	T_{frac}	0.500	1.000	drop — redundant with M_{frac} ($\rho \approx 0.949$)
vector-depth family	VD	—	—	supporting chart only
frequency lattice	FL	—	—	supporting chart only
source-native collective evidence	C_{coll}	—	—	physical extension, not a temporal gate

8.3 Source anchor

$$P_{src}: \quad p_{\text{phase-label}} \leq 0.10.$$

No raw parent-magnitude threshold — P_{src}'s raw magnitude is not discriminative on its own (Malz–Smith topological QP has raw-magnitude AUC 0.125, i.e. inverted; Table 2), which is exactly why P_{src} is retained only as a null-significance gate.

8.4 Chart coordinate

J_{frac} is mandatory output but non-gating (§5).

8.5 Primary temporal decision coordinate

$$M_{frac} \geq 0.12 \quad \text{and} \quad p_{\text{Fourier-phase}} \leq 0.10,$$

with the raw M_{frac} condition required to survive every frozen robustness transform (§7).

8.6 Collective extension

C_{coll} remains a separate, domain-specific physical-evidence interface (`COLLECTIVE_NOT_ASSESSED` / `COLLECTIVE_EVIDENCE_UNAVAILABLE` / `COLLECTIVE_EVIDENCE_AVAILABLE_FOR_DOMAIN_SPECIFIC_REVIEW`); no universal numerical collective gate is invented (§12).

8.7 Frozen state precedence ladder

First match wins:

1. OUTSIDE_CHART_DOMAIN — not a genuine two-clock time-domain record.
2. OUTSIDE_EMPIRICALLY_QUALIFIED_RATIO_DOMAIN — source ratio outside the golden-ratio branch (v001).
3. SOURCE_UNANCHORED — fails the P_{src} phase-label-permute null ($p_{\text{upper}} > 0.10$).
4. MIXED_ORGANIZATION_WEAK — M_{frac} below the raw gate 0.12.

5. MIXED_ORGANIZATION_SPECTRAL_NULL — fails the M_{frac} Fourier-phase null ($p_{\text{upper}} > 0.10$).
6. MIXED_ORGANIZATION_UNSTABLE — raw gate fails to hold under one or more of the six stability transforms.
7. TEMPORAL_CORE_SUPPORTED — P_{src} anchored, $M_{frac} \geq 0.12$, Fourier-null passed, stable under all six transforms.

Development replay at freeze. 4/4 DTQC positives reach TEMPORAL_CORE_SUPPORTED; 0/2 Luo LOW, 0/2 Luo HIGH, 0/6 Malz–Smith topological, and 0/6 Malz–Smith trivial controls do (0/16 overall). All sixteen controls fail through SOURCE_UNANCHORED or MIXED_ORGANIZATION_WEAK; no post-hoc rule was added to force this separation, and it follows directly from the frozen P_{src}/M_{frac} gates.

9 Prospective P001 Falsification

This section is given substantial prominence deliberately: it is not a minor validation subsection but the main evidence that the frozen grammar was genuinely falsifiable.

9.1 Pre-registration

An external-model transfer based on the Marripour–Abouie [12] theoretical model family (an $L = 4$ periodic chain, $J = 5.5$, $h = 0.3$, 100 paired disorder realizations, initial $|+x\rangle^{\otimes L}$, ensemble-averaged $m_x(t)$, 64 drive periods, 16 samples/period, $\Omega/\omega_d = 1/\varphi$ — deliberately adapted to the golden-ratio domain rather than the model's originally published ratio). Two frequency-scale conditions were pre-registered and generated blind, under cryptographic mapping commitment: $\omega_d = 12$ (candidate hypothesis: long-lived prethermal quasiperiodic order, predeclared TEMPORAL_CORE_SUPPORTED) and $\omega_d = 1$ (control hypothesis: rapid heating / loss of order, predeclared not TEMPORAL_CORE_SUPPORTED). Blind labels `P001_A`, `P001_B` were assigned before any grammar evaluation.

9.2 Blind result

After lock and before reveal: one record returned SOURCE_UNANCHORED; the other returned MIXED_ORGANIZATION_WEAK.

9.3 Reveal

Table 7: Prospective P001 result. Blind evaluation and locking preceded identity reveal.

Blind ID	Revealed identity	Predeclared	Coordinates	State	Met?
`P001_A`	`LOW_FREQ_BREAKDOWN_CONTROL_HYPOTHESIS` (ω_d=1)	not TEMPORAL_CORE_SUPPORTED	P_{src} p=0.14; J_{frac}=−0.038; M_{frac}=−0.014; Fourier p=0.60	SOURCE_UNANCHORED	**yes**
`P001_B`	`HIGH_FREQ_CANDIDATE_HYPOTHESIS` (ω_d=12)	TEMPORAL_CORE_SUPPORTED	P_{src} p=0.01; J_{frac}=−0.166; M_{frac}=−0.330; Fourier p=1.00	MIXED_ORGANIZATION_WEAK	**no**

The high-frequency candidate — the record predeclared to show sustained multi-clock order — was the rejected record. We report its values prominently because this was not a threshold near-miss:

$$P_{src} : p = 0.01, \qquad J_{frac} \approx -0.166, \qquad M_{frac} \approx -0.330, \qquad p_{\text{Fourier}} = 1.00.$$

$M_{frac} \approx -0.330$ is the opposite sign from every development-stage DTQC-positive record, and its worst robustness variant (`PREFIX_0.75`) drove it to $M_{frac} \to -1.0245$ — moving further from, not toward, the gate. The control behaved exactly as predeclared (worst variant `DOWNSAMPLE_5`, $M_{frac} \to -0.2024$).

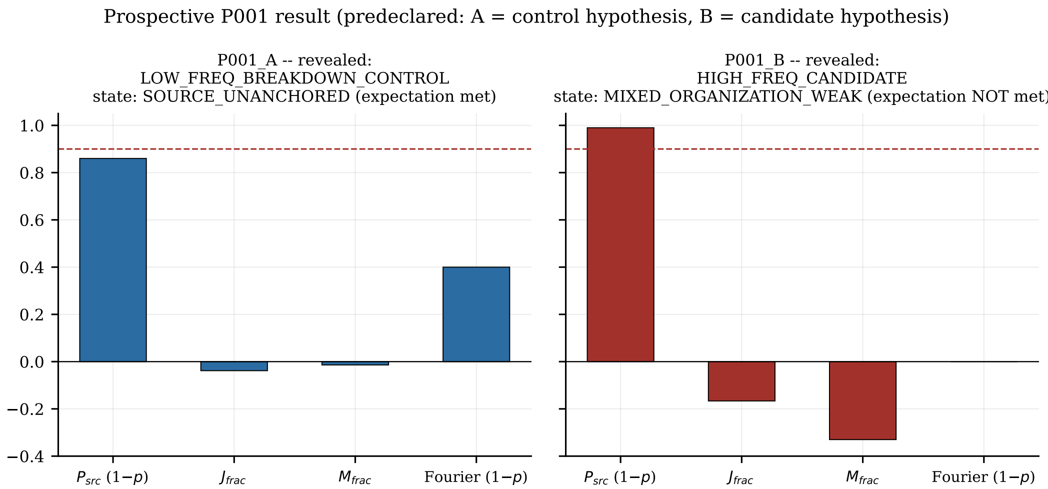

Figure 3: Prospective P001 result. Bars show $1 - p$ for the two null gates (so the dashed line at 0.90 marks the frozen $p \leq 0.10$ pass threshold on a common "higher is better" axis) alongside the raw J_{frac} and M_{frac} values. `P001_A` (left) matched its predeclared control hypothesis. `P001_B` (right) failed its predeclared candidate hypothesis: its source anchor *passes* ($P_{src}\, p = 0.01$) while its fractional organization is strongly negative, the reverse of the DTQC development pattern.

9.4 Interpretation

Formal outcome: `PROSPECTIVE_FAILURE_CANDIDATE_REJECTED`. **[demonstrated prospectively]** No parameter of `MC_GRAMMAR_v001` was modified after this result. Per the pre-registered protocol, this failure cannot be repaired by changing thresholds, nulls, robustness transforms, the ratio domain, the representation, or the mapping after the fact; any repair requires declaring `MC_GRAMMAR_v002` or later. This does not itself disprove the external model's predicted quasiperiodic/prethermal order — it shows only that the frozen grammar did not admit this pre-registered trajectory. **[not claimed]** `MC_GRAMMAR_v001` is a universal multi-clock temporal grammar — this claim is directly contradicted by this result, and we do not make it anywhere in this manuscript.

10 c003 — The Quarantined Historical Test

Only now does the boundary case that motivated this entire branch reenter the analysis.

10.1 Historical integer-grammar result

Under Manuscript I's frozen integer-depth grammar (unchanged, reproduced here for reference):

$$q_0 = 6, \qquad C \approx 0.1988, \qquad F \approx 0.0934, \qquad p \approx 0.9602, \qquad \text{verdict NO\_TEMPORAL\_ORDER.}$$

Correctly interpreted: c003 was outside the supported domain of the integer-depth grammar, not shown to lack temporal order.

10.2 Final multi-clock adapter

The original event-time trajectory was used [6], Fig. 1d as candidate and Fig. 1b as its matched breakdown control, with documented source ratio $\tau_2/\tau_1 = 1.618$ and source coordinate $x = t_{\tau_1}/1.618$. No interpolation, filtering, sign alignment, or favorable sub-windowing was applied.

10.3 Side-by-side result

Table 8: C003 versus its matched breakdown control under the frozen MC_GRAMMAR_v001, evaluated once, after the P001 reveal.

Quantity	C003	Matched control
P_{src} (p)	**1.000**	**0.010**
J_{frac}	**+0.910688**	**−0.593846**
M_{frac}	**+0.596092**	**−0.003958**
Fourier-null p	**0.020**	0.780
Robust min M_{frac}	**+0.595093**	−0.558942
Frozen state	SOURCE_UNANCHORED	MIXED_ORGANIZATION_WEAK

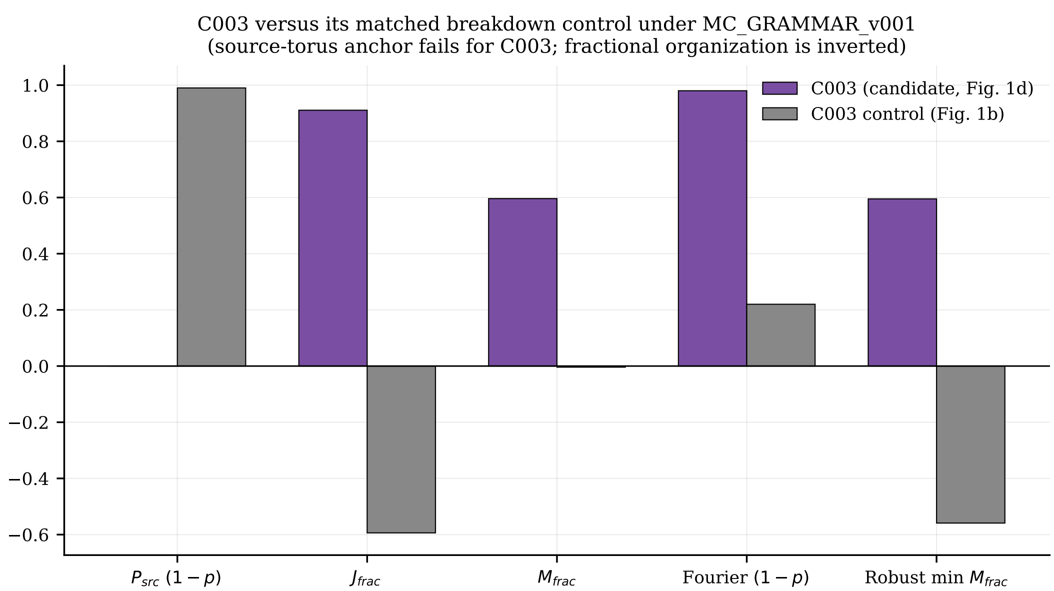

Figure 4: C003 (violet) versus its matched breakdown control (gray) on every frozen MC_GRAMMAR_v001 coordinate. Null gates are shown as $1-p$ so that both bars read "higher is better." C003's source anchor fails outright ($1-p=0$) while its fractional multi-clock coordinates — the JPR-derived J_{frac} and the FC-derived M_{frac} — are the strongest seen anywhere in this study, with M_{frac} additionally the most robust — stable across all six frozen transforms (rightmost pair of bars); the control shows close to the opposite pattern.

Formal result.

$$\boxed{\text{C003} \notin D(\texttt{MC\_GRAMMAR\_v001})}$$

The hoped-for simple result, c003 $\in D(G_{\text{multi}})$, is therefore **not established**. c003 remains formally unadmitted — `C003_REMAINS_OUTSIDE_FROZEN_MULTI_CLOCK_GRAMMAR` — because state precedence checks P_{src} before M_{frac}, and P_{src} fails at the worst possible value ($p = 1.000$). This is not a positive admission of c003; it is a sharper diagnosis of *why* the historical boundary persists, developed fully in §11.

11 The Parent/Fractional Inversion

11.1 c003

$$P_{src} \text{ fails}, \qquad \text{while} \qquad J_{frac} \gg 0, \qquad M_{frac} \gg 0.$$

11.2 Matched control

$$P_{src} \text{ passes}, \qquad \text{while} \qquad J_{frac} < 0, \qquad M_{frac} \approx 0.$$

11.3 Structural interpretation

`MC_GRAMMAR_v001` implicitly assumes a hierarchy,

$$\text{parent/source anchoring} \to \text{fractional organization},$$

visible directly in its state-precedence ladder (§8), which checks P_{src} before M_{frac} can even be consulted. Every development-stage DTQC-positive record is consistent with this hierarchy: source anchor holds, and fractional organization follows. c003 does not fit it. It exhibits

$$\boxed{\text{weak parent anchoring} \;+\; \text{strong fractional organization.}}$$

We provisionally name this pattern the *fractional-primary candidate regime*. We do not yet claim a universal physical class: **[not claimed]** fractional-primary organization is an established structural mode. The data establish a structural pattern in one candidate/control pair, not its universal ontological status. Figure 5 renders the two organizational paths side by side.

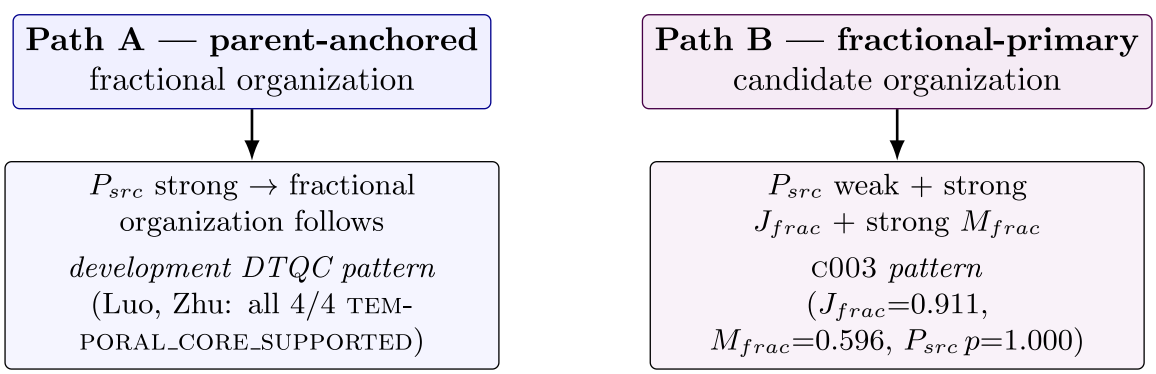

Figure 5: Two organizational paths within multi-clock temporal order, conceptual synthesis without claiming the two paths are already universal physical species. Path A is the pattern the frozen grammar was built around and the pattern every development-stage DTQC-positive record follows. Path B is the pattern c003 alone exhibits in this study.

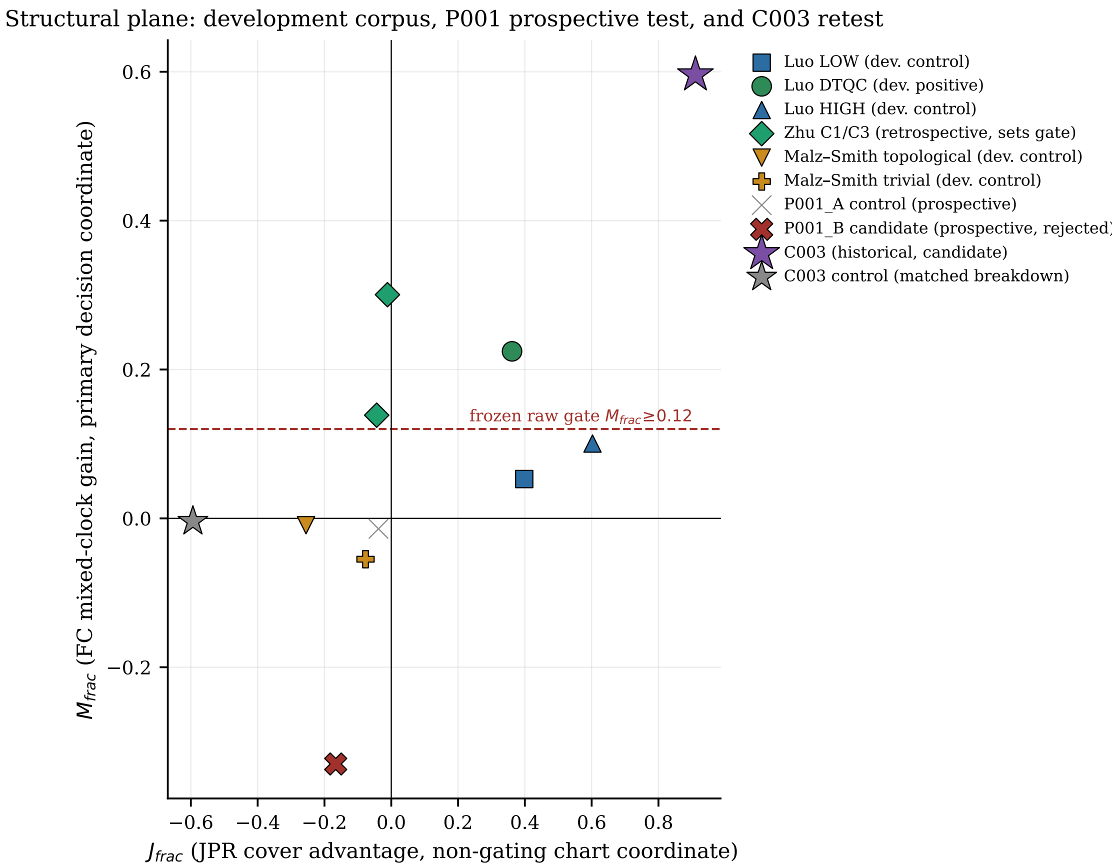

Figure 6: The (J_{frac}, M_{frac}) structural plane: development corpus (Luo, Zhu, Malz–Smith), the P001 prospective candidate/control, and C003 with its matched control. The frozen raw gate $M_{frac} \geq 0.12$ is shown as a dashed line. C003 (large violet star) sits far outside the cluster of every other record in this study, on both axes simultaneously; its control (gray star) sits close to the Malz–Smith controls despite arising from a physically distinct source. The rejected P001_B candidate (red $\times$) sits with the controls, not with the development-stage positives.

12 Temporal Organization vs Collective Coupling

C_{coll} was deliberately kept outside the temporal decision grammar (§8); the Luo regimes show why. Temporal mixed-clock organization approximately follows

$$M_{frac}: \quad \text{DTQC} > \text{HIGH} > \text{LOW},$$

while collective entanglement approximately follows the reverse,

$$S: \quad \text{LOW} > \text{DTQC} > \text{HIGH}.$$

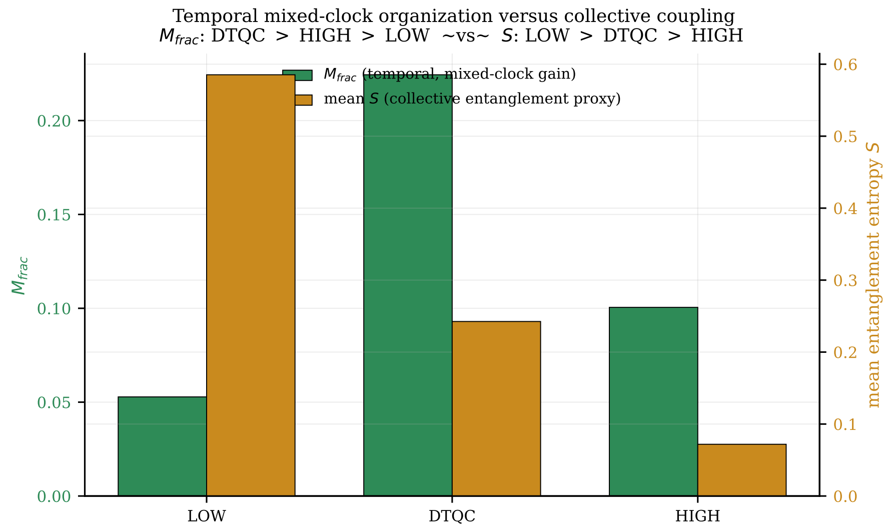

Figure 7: Temporal mixed-clock organization (M_{frac}, green, left axis) versus collective entanglement (S, amber, right axis) across the three Luo regimes. The two axes are not monotonically related: high-frequency decoupling reduces collective coupling while retaining substantial temporal fractional structure.

Therefore

$$\boxed{\text{temporal multi-clock organization} \neq \text{collective coupling.}}$$

This is not a philosophical separation imposed by UNNS terminology; it is demanded by the observed regime ordering, and it echoes the temporal/collective distinction already established for the integer-depth grammar in Manuscript I's four-sector hierarchy.

13 From Chart Domains to Structural Strata

Manuscript I suggested

$$\text{UNNS} = \text{structural atlas} + \text{charts} + \text{chart domains}.$$

The present work suggests an extension:

$$\boxed{\text{UNNS} = \text{charts} + \text{chart domains} + \text{internal structural strata} + \text{transition relations.}}$$

The new empirical motivation is that even within a legitimate two-clock chart, temporal organization may enter through different structural hierarchies. A possible distinction: parent-anchored fractional organization (Path A, §11) versus fractional-primary candidate organization (Path B). We treat this as a research hypothesis generated by the data, not as a completed theorem — the same posture Manuscript I took toward its own then-undeveloped multi-clock coordinates, now one level further in.

14 Limitations and the Next Grammar

14.1 Hard limitations

1. `MC_GRAMMAR_v001` is empirically qualified only on the golden-ratio branch.
2. There is only one completed prospective candidate/control pair (P001); a single blind pair is not a statistically powered prospective campaign.
3. That prospective positive candidate failed.
4. Zhu is formative/development evidence, not independent final validation.
5. Malz–Smith is a pre-freeze specificity challenge later incorporated into development.
6. No universal numerical C_{coll} gate exists.
7. C003 is not admitted by the frozen grammar.
8. The proposed fractional-primary interpretation is based on a strong candidate/control structural inversion in a single case, and is not yet a universally established physical class.

14.2 Next scientific question

> **Is parent-anchored multi-clock order structurally distinct from fractional-primary multi-clock order?**

Operationally: can a new grammar architecture that does not require parent anchoring recover the C003-like fractional-primary pattern in retrospective stress tests while continuing to reject the already-known P001 candidate and development controls — and then survive genuinely new, unseen prospective candidate/control data?

Critical firewall for future work. C003 and P001 are now consumed evidence. They may serve as retrospective stress tests for a future grammar. They can never again serve as prospective validation. Any new architecture must be `MC_GRAMMAR_v002` or later — never a silent revision of v001.

15 Conclusion — A Deeper Boundary Than Expected

Manuscript I's methodological promise implied an intended simple sequel: either C003 $\notin D(G_{\text{integer}})$ stays unresolved, or perhaps C003 $\in D(G_{\text{multi}})$ once a multi-clock grammar exists. That is not what happened. Instead,

$$\text{C003} \notin D(G_{\text{integer}}), \qquad \text{C003} \notin D(\texttt{MC\_GRAMMAR\_v001}),$$

yet

$$\boxed{\text{C003 exhibits exceptionally strong fractional multi-clock structure}}$$

that is absent from its matched breakdown control, with its M_{frac} component stable under every frozen robustness transform. Meanwhile, the grammar had already failed a prospective positive transfer test (P001) before C003 was ever reopened — the falsifiability this manuscript's title claims for it was exercised, not merely asserted.

The manuscript therefore ends with the real scientific gain:

> **The experiment did not validate a universal detector. It exposed a deeper structural boundary inside multi-clock temporal organization itself.**

This advances grammar-relative structural visibility — introduced conceptually in Manuscript I §11 — from a conceptual interpretation to an empirically constrained, falsifiable research program: a frozen grammar's own architecture (parent anchoring as a precondition for fractional organization) can itself be shown, by a single sharply diagnostic case, to be a hypothesis rather than a law, and the next grammar version inherits the obligation to test that hypothesis prospectively rather than retrofit it to C003.

Data and Code Availability

The frozen grammar (`MC_GRAMMAR_v001`, SHA-256 `8ae9b0d9...39e4bba556d6`), the 13-record neutral-ingestion corpus, the 27-record representation-study corpus, the pre-freeze Malz–Smith specificity lock, the locked Zhu v002 transfer-test record, all P001 prospective locks (blind analysis, quantitative lock, mapping-commitment verification), and the C003 final-evaluation audit are maintained under the `UNNS_MULTI_CLOCK_RECURRENCE` branch tree, alongside `RESEARCH_FIREWALL.md`, `CORPUS_v001.json`, `CORPUS_STATUS_v002.json`, `GRAMMAR_FREEZE_v001/README.md`, and `C003_FINAL_v001/C003_FINAL_AUDIT.json`. Firewall discipline by stage is: C003 not loaded during grammar development, selection, or freeze; C003 not loaded during the P001 blind analysis or reveal; thresholds and nulls unchanged throughout. C003 was loaded exactly once, for its final evaluation, only after the P001 reveal. As in Manuscript I, this internal archive is not yet deposited at a public, citable repository; a versioned public archive will be deposited at a permanent repository at the time of external submission, and this section updated with the corresponding DOI.

External-data provenance. The physical sources analyzed in this branch (internally labeled Huang, Luo, Zhu, Malz–Smith, and the Marripour–Abouie theoretical-model family used for P001) are tracked in `CORPUS_v001.json` and `CORPUS_STATUS_v002.json` with per-record role and status history, and are now fully cited against their published external record (References, below): Huang [8], Luo [9], Zhu [10], Malz–Smith [11], and Marripour–Abouie [12]. Malz–Smith is a quasiperiodic, multi-frequency topological-Floquet-lattice experiment on a single superconducting qubit — not itself a discrete-time-crystal report; it enters this branch only as a pre-freeze specificity control (§7), and is cited and described here on those terms rather than as a DTQC source. One reconciliation item remains open before external submission: Luo's own published Data Availability statement points to Zenodo DOI `10.5281/zenodo.18396451`, which has not yet been cross-checked against this branch's internal project record number for the same dataset; until that check is complete, the internal record and the published Zenodo archive should be treated as provisionally, not confirmedly, identical. No other external-citation item remains open; this paragraph replaces the earlier provenance-pending language carried in prior drafts of this manuscript.

References

[1] F. Wilczek, "Quantum Time Crystals," *Phys. Rev. Lett.* **109**, 160401 (2012).

[2] D. V. Else, B. Bauer, and C. Nayak, "Floquet Time Crystals," *Phys. Rev. Lett.* **117**, 090402 (2016).

[3] N. Y. Yao, A. C. Potter, I.-D. Potirniche, and A. Vishwanath, "Discrete Time Crystals: Rigidity, Criticality, and Realizations," *Phys. Rev. Lett.* **118**, 030401 (2017).

[4] K. Sacha and J. Zakrzewski, "Time Crystals: A Review," *Rep. Prog. Phys.* **81**, 016401 (2018).

[5] M. P. Zaletel, M. Lukin, C. Monroe, C. Nayak, F. Wilczek, and N. Y. Yao, "Colloquium: Quantum and Classical Discrete Time Crystals," *Rev. Mod. Phys.* **95**, 031001 (2023).

[6] G. He, B. Ye, R. Gong, C. Yao, Z. Liu, K. W. Murch, N. Y. Yao, and C. Zu, "Experimental Realization of Discrete Time Quasicrystals," *Phys. Rev. X* **15**, 011055 (2025). [Source of the c003 candidate and matched control analyzed in §10; raw data via He et al./Zenodo record 14791637.]

[7] UNNS Research Collective, *Prospective Structural Classification of Discrete Time-Crystalline Recurrence: Integer-Depth Closure, Blind Validation, and a Quasi-Periodic Domain Boundary* (Manuscript I / TIME-CRYSTAL-I v1.1.0), UNNS Substrate Research Program (2026). [The companion volume this manuscript deliberately does not extend or amend; see §1–§2.]

[8] Huang *et al.*, "Observation of Continuous Time Crystals and Quasi-Crystals in Spin Gases," *Nature Communications* **16**, 9375 (2025), DOI: 10.1038/s41467-025-64413-y. Source data supplied with the article; internal project record `HUANG_2025` (§3).

[9] Luo *et al.*, "Discrete Time Quasi-Crystals in Rydberg Atomic Chain," *Communications Physics* **9**, 141 (2026), DOI: 10.1038/s42005-026-02572-0. Data Availability: Zenodo 10.5281/zenodo.18396451; reconciliation with the internal project record `LUO_2026` (§3, §5, §12) is an open item, see Data and Code Availability.

[10] Zhu *et al.*, "Observation of Discrete Time Quasicrystal in Rydberg Atomic Gases," *Nature Communications* (2026), DOI: 10.1038/s41467-026-75966-x. Dataset: Zenodo 10.5281/zenodo.20698244; internal project record `ZHU_2026` (§3.1, §5, §6).

[11] D. Malz and A. Smith, "Topological Two-Dimensional Floquet Lattice on a Single Superconducting Qubit," *Phys. Rev. Lett.* **126**, 163602 (2021), DOI: 10.1103/PhysRevLett.126.163602. Code/data: Zenodo 10.5281/zenodo.4560106. A quasiperiodic, multi-frequency Floquet-lattice experiment on a single superconducting qubit — not itself a discrete-time-crystal report; used in this branch only as a pre-freeze specificity control (§7), internal archive `qubit-topological-floquet-v2.0.zip`.

[12] M. Marripour and J. Abouie, "Emergence of Prethermal Time-Quasicrystalline Order in a Quasiperiodically Driven Noninteracting Spin Chain," *Phys. Rev. B* **113**, 214318 (2026), DOI: 10.1103/317d-p5fb. External theoretical-model Hamiltonian family used for the P001 prospective transfer test (§9).