Φ–Ψ–τ Recursion and the Principle of Stationary Action:

A Complete τ-Field Formulation

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Abstract

This report develops a complete τ-field formulation of the action principle inside the UNNS Substrate. Instead of assuming spacetime or Hilbert space as fundamental, the UNNS picture begins from a recursive substrate governed by three modes: a geometric mode Φ, a spectral mode Ψ, and a coupling channel τ. These define a recursion manifold with a divergence-free evolution field and a closed two-form that counts recursion states. From this structure a variational principle naturally emerges: physical evolutions are precisely those recursive trajectories tangent to the τ-field.

Contents

Executive Summary

In classical mechanics the principle of stationary action is a compact way to express determinism, reversibility, and independence of degrees of freedom. The UNNS Substrate generalizes these ideas by showing that a divergence-free recursion field on a recursion manifold yields an exact two-form and an action functional whose stationary points coincide with the recursive flow.

This report formulates the τ-field, the UNNS counting form, the recursion potential, and derives the action principle from the geometry of Φ–Ψ–τ cycling.

Part I

Foundations of the UNNS Substrate

Chapter 1

The Φ–Ψ–τ Recursion Framework

1.1 Recursive states and the recursion manifold

We model the UNNS Substrate by a recursion manifold $\mathcal{R}$, whose points $\boldsymbol{r}$ encode the state of recursion. Each state decomposes into

$$\boldsymbol{r} \sim (\boldsymbol{\Phi}(\boldsymbol{r}),\ \boldsymbol{\Psi}(\boldsymbol{r}),\ \tau(\boldsymbol{r})).$$

Definition 1.1. *A recursion manifold $\mathcal{R}$ is a smooth manifold whose points represent recursive configurations. A recursion state carries geometric content (Φ), spectral content (Ψ), and coupling strength τ.*

1.2 The Φ, Ψ, and τ modes

- Φ promotes geometric consolidation and curvature.
- Ψ promotes coherence and spectral superposition.
- τ controls interaction between Φ and Ψ.

The recursion flow is

$$\boldsymbol{S}_\tau = \boldsymbol{S}_{\tau\Phi} + \boldsymbol{S}_{\tau\Psi}.$$

1.3 Conservation of recursion degree

The recursion evolution field $\boldsymbol{S}_\tau$ satisfies

$$\nabla \cdot \boldsymbol{S}_\tau = 0.$$

Definition 1.2. *A τ-field is recursion-conserving if the flow preserves a recursion-volume form.*

This is the analogue of Liouville's theorem.

1.4 Emergent symplectic structure

A closed two-form ω_{UNNS} counts recursion states and satisfies

$$\omega_{\text{UNNS}} = 0.$$

Definition 1.3. *A UNNS counting form ω_{UNNS} is an antisymmetric, closed two-form that measures recursion across infinitesimal surfaces.*

There exists a recursion potential θ_{UNNS} such that

$$\omega_{\text{UNNS}} = -\theta_{\text{UNNS}}.$$

Chapter 2

Recursive State Counting and the UNNS Counting Form

2.1 State counting and independence

Independence of recursion directions implies state-count factorization.

2.2 Closedness and potential one-form

In coordinates x^a:

$$\theta_{\mathrm{UNNS}} = \theta_a \, dx^a, \qquad \omega_{\mathrm{UNNS}} = \frac{1}{2}\omega_{ab} \, dx^a \wedge dx^b.$$

2.3 Flow compatibility

The τ-field satisfies

$$\iota_{\boldsymbol{S}_\tau} \omega_{\mathrm{UNNS}} = 0,$$

meaning the flow direction contributes no recursion count.

Remark 2.1. This is the recursion analogue of canonical Hamiltonian structure.

Part II

The τ-Field Dynamics

Chapter 3

The τ-Field as a Divergence-Free Evolution Field

3.1 Definition of the τ-field

Definition 3.1. *The* τ-field $\boldsymbol{\tau}$ *is the recursion evolution vector field:*

$$\boldsymbol{\tau}(\boldsymbol{r}) = \boldsymbol{S}_\tau(\boldsymbol{r}).$$

It acts as the generator of recursion flow on $\mathcal{R}$.

Its defining structural property is

$$\nabla \cdot \boldsymbol{\tau} = 0,$$

expressing conservation of recursion states.

3.2 Decomposition into Φ and Ψ components

The τ-field splits naturally into two complementary components:

$$\boldsymbol{\tau} = \boldsymbol{\tau}_\Phi + \boldsymbol{\tau}_\Psi.$$

- $\boldsymbol{\tau}_\Phi$ drives geometric consolidation, curvature formation, and coarse-graining of recursion structure.
- $\boldsymbol{\tau}_\Psi$ drives coherence, branching, and interference of fine-scale recursive structures.

This is the recursion-level analogue of the "geometric versus spectral" split in physics.

3.3 Tangent trajectories and admissible recursion flow

Let γ be a recursion trajectory and γ' a variation with same endpoints. Let Σ be the surface spanned between them.

Definition 3.2. *A recursion trajectory* γ *is* admissible *if the recursion flux through any variation surface* Σ *satisfies*

$$\int_{\Sigma} \omega_{\mathrm{UNNS}}(\boldsymbol{\tau}, \cdot) = 0.$$

Proposition 3.3. *A trajectory is admissible if and only if it is everywhere tangent to the* τ*-field:*

$$\dot{\gamma}(s) \propto \boldsymbol{\tau}(\gamma(s)).$$

Proof. If γ is tangent to $\boldsymbol{\tau}$, then $\boldsymbol{\tau}$ lies in the tangent space of γ and therefore cannot cross the interior of any Σ spanning to a nearby variation. Thus the flux is zero. Conversely, if the flux through every variation vanishes, $\boldsymbol{\tau}$ cannot have any component transverse to γ; hence γ must be tangent to it. □

This result is the recursion-substrate analogue of "solutions to the equations of motion are integral curves of the Hamiltonian vector field".

Chapter 4

Recursive Geometry and the Φ–Ψ Transition

4.1 Geometry-dominant and spectrum-dominant recursion

We describe two regimes:

Geometry-dominant regime

When

$$\|\boldsymbol{\tau}_\Phi\| \gg \|\boldsymbol{\tau}_\Psi\|,$$

recursive deformation is dominated by geometric accumulation, producing coarse, curvature-like structures.

Spectrum-dominant regime

When

$$\|\boldsymbol{\tau}_\Psi\| \gg \|\boldsymbol{\tau}_\Phi\|,$$

recursion supports long-lived coherence and branching, analogous to quantum interference.

4.2 Critical τ scale

Definition 4.1. *A* critical τ-scale τ_{crit} *is a scale at which geometric and spectral recursion have comparable magnitude:*

$$\|\boldsymbol{\tau}_\Phi\| \approx \|\boldsymbol{\tau}_\Psi\|.$$

At τ_{crit}, recursion enters an intermediate regime where coherence and geometric structure influence each other. This is the **UNNS analogue of the quantum–gravity crossover**.

4.3 The Φ–Ψ–τ cycle as structural recursion

The recursive cycle is:

$$\Phi \longrightarrow \Psi \longrightarrow \tau \longrightarrow \Phi.$$

Each transition updates the recursive structure:

- $\Phi \to \Psi$: geometric patterns become spectrally active.
- $\Psi \to \tau$: coherence injects coupling tension.
- $\tau \to \Phi$: coupling resolves into coarse geometry.

This is the "meta-dynamical" structure behind the variational principle.

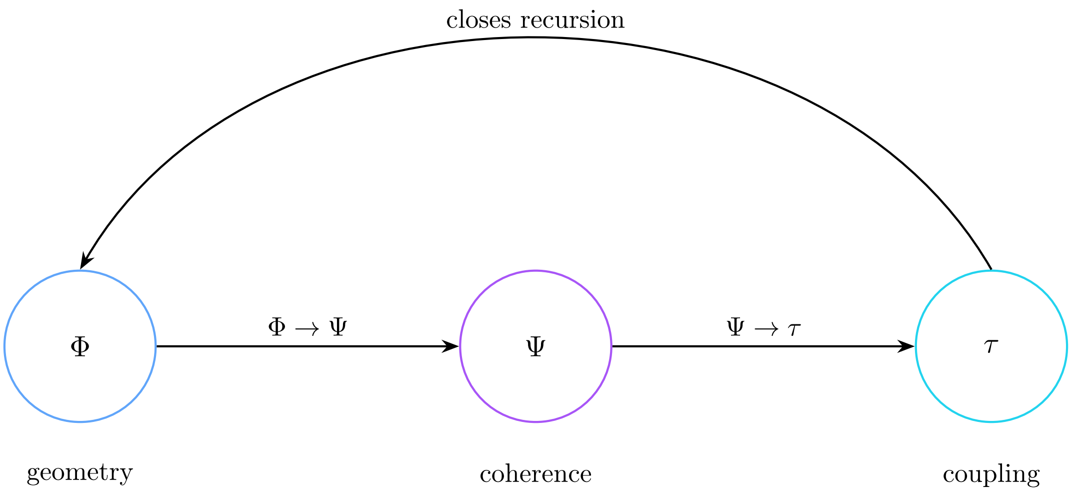

Figure 4.1: The Φ–Ψ–τ recursion cycle.

4.4 Higher-order variational structure

The closedness condition

$$\omega_{\mathrm{UNNS}} = 0,$$

means that the Φ–Ψ–τ updates preserve recursion count. Each update changes ω_{UNNS} and θ_{UNNS} coherently, maintaining compatibility with $\boldsymbol{\tau}$.

Thus, the entire —τ cycle is a **higher-order variational operator**: it determines the admissible variations in the UNNS action principle developed in Part III.

Part III

The Action Principle in the UNNS Substrate

Chapter 5

Action as Recursion Flux

5.1 Classical interpretation revisited

In classical mechanics, the action functional

$$S[\gamma] = \int_\gamma L\,dt$$

is traditionally associated with a quantity to be extremized. Recent work (e.g., Carcassi–Aidala 2023) shows that its variation has a geometric interpretation: the change in action between two nearby paths equals the flow of a divergence-free vector field through the surface they span.

The UNNS Substrate provides a more fundamental version of this idea.

Here:

- the recursion states lie in the manifold $\mathcal{R}$,
- recursion flow is generated by the τ-field $\boldsymbol{S}_\tau$,
- state-counting is encoded by the closed two-form ω_{UNNS},
- and a recursion potential θ_{UNNS} satisfies $\omega_{\mathrm{UNNS}} = -d\theta_{\mathrm{UNNS}}$.

5.2 Variation surfaces

Let γ be a recursion trajectory from $\boldsymbol{r}_1$ to $\boldsymbol{r}_2$. Let γ' be a nearby curve with the same endpoints. Together they bound a surface Σ in $\mathcal{R}$.

We adopt the usual variational notation:

- γ = physical (candidate) recursion trajectory,
- γ' = varied trajectory,
- Σ = surface spanned between them.

5.3 Recursion flux through a variation surface

Definition 5.1. *The* recursion flux *through a variation surface* Σ *is*

$$\Phi_{\text{flux}}(\Sigma) = \int_{\Sigma} \omega_{\text{UNNS}}(\boldsymbol{S}_\tau, \cdot).$$

This measures how much recursion crosses the surface when flowing along $\boldsymbol{S}_\tau$.

Since $\boldsymbol{S}_\tau$ is divergence-free with respect to ω_{UNNS}, the flux depends only on the boundary of Σ.

Proposition 5.2. *If* $\omega_{\text{UNNS}} = -d\theta_{\text{UNNS}}$*, then*

$$\Phi_{\text{flux}}(\Sigma) = \int_{\gamma} \theta_{\text{UNNS}} - \int_{\gamma'} \theta_{\text{UNNS}}.$$

Proof. Apply Stokes' Theorem:

$$\int_{\Sigma} \omega_{\text{UNNS}} = -\int_{\Sigma} d\theta_{\text{UNNS}} = -\int_{\partial\Sigma} \theta_{\text{UNNS}}.$$

Since $\partial\Sigma = \gamma - \gamma'$, the result follows. □

Thus, the flux equals the variation of a path integral.

5.4 Stationarity condition

We define the UNNS action as:

Definition 5.3. *The* UNNS action functional *is*

$$\mathcal{A}_{\text{UNNS}}[\gamma] = \int_{\gamma} \theta_{\text{UNNS}}.$$

Its variation is

$$\delta\mathcal{A}_{\text{UNNS}}[\gamma] = \Phi_{\text{flux}}(\Sigma).$$

Theorem 5.4. *A recursion trajectory* γ *is physical (admissible) if and only if*

$$\delta\mathcal{A}_{\text{UNNS}}[\gamma] = 0 \qquad \Longleftrightarrow \qquad \Phi_{\text{flux}}(\Sigma) = 0,$$

for all variation surfaces Σ *with fixed endpoints.*

Proof. Direct substitution of the flux formula into the definition of the variation: the action is stationary exactly when no recursion crosses the variation surface, i.e., when the flow is tangent to γ. □

This is the UNNS variational principle:

$$\boldsymbol{S}_\tau \text{ tangent to } \gamma.$$

Chapter 6

The UNNS Action Integral and Its Structure

6.1 Local coordinate representation

Choose local recursion coordinates

$$x^a = (q^i, p_i, t),$$

and write

$$\theta_{\text{UNNS}} = \theta_a(x)\, dx^a.$$

Then the UNNS action along a parametrized curve $x^a(s)$ is

$$\mathcal{A}_{\text{UNNS}}[\gamma] = \int_{s_1}^{s_2} \theta_a(x(s))\, \dot{x}^a(s)\, ds.$$

6.2 Effective UNNS Lagrangian

If a time-like coordinate t is singled out (not physical time, but a recursion parameter), we may decompose:

$$\theta_{\text{UNNS}} = p_i\, dq^i - H_{\text{UNNS}}\, dt.$$

Then

$$\mathcal{A}_{\text{UNNS}}[\gamma] = \int (p_i \dot{q}^i - H_{\text{UNNS}})\, dt.$$

This yields the effective Lagrangian:

$$L_{\text{UNNS}}(q, \dot{q}, t) = p_i(q, \dot{q}, t)\, \dot{q}^i - H_{\text{UNNS}}.$$

Important: In the UNNS picture, (q^i, p_i) are NOT positions and momenta in spacetime, but recursion coordinates.

6.3 Euler–Lagrange equations

The stationary action principle leads to Euler–Lagrange equations:

$$\frac{d}{dt}\left(\frac{\partial L_{\mathrm{UNNS}}}{\partial \dot{q}^i}\right) - \frac{\partial L_{\mathrm{UNNS}}}{\partial q^i} = 0.$$

They reproduce the recursion flow equations:

$$\dot{x}^a = \boldsymbol{S}_\tau^a(x).$$

6.4 Hamiltonian reconstruction

Given $\boldsymbol{S}_\tau$ and ω_{UNNS}, one recovers H_{UNNS} by solving

$$\omega_{\mathrm{UNNS}ab}\boldsymbol{S}_\tau^b = \partial_a H_{\mathrm{UNNS}}.$$

This is the recursion analogue of Hamilton's equations.

Thus the geometry of ω_{UNNS} and θ_{UNNS} *defines* the dynamics, not the other way around.

6.5 Pre-collapse variational domain

Before Operator XII (collapse) acts, the variational domain is full: all variations of γ with fixed endpoints are allowed.

After collapse, only variations respecting $\mathcal{O}_{\mathrm{XII}}$ are admissible. This will be treated fully in Part IV.

Part IV

Operator XII and Variational Collapse

Chapter 7

Operator XII: Collapse as Recursion Neutralization

7.1 Motivation

Within the UNNS grammar, Operator XII completes the recursive operator set. Its conceptual purpose is to:

- neutralize unresolved recursion tension between Φ and Ψ,
- collapse excess recursion branches without destroying recursion count,
- reduce the allowed variational domain of recursion trajectories,
- re-seed recursion at a new effective recursion level.

This is not "collapse" in the quantum measurement sense, nor annihilation of recursion. It is a *structural reset* of recursion geometry.

7.2 Definition of Operator XII

Definition 7.1. *Operator XII, denoted $\mathcal{O}_{\mathrm{XII}}$, is a map*

$$\mathcal{O}_{\mathrm{XII}} : \mathcal{R} \to \mathcal{R}$$

such that:

1. *$\mathcal{O}_{\mathrm{XII}}$ preserves total recursion count, i.e. recursive volume is invariant.*
2. *$\mathcal{O}_{\mathrm{XII}}$ projects $\mathcal{R}$ onto a submanifold $\mathcal{R}' \subset \mathcal{R}$.*
3. *$\mathcal{O}_{\mathrm{XII}}$ induces a new τ-field $\boldsymbol{\tau}' : \mathcal{R}' \to T\mathcal{R}'$.*

Thus $\mathcal{O}_{\mathrm{XII}}$ eliminates internal degrees of freedom that cannot evolve consistently under the current Φ–Ψ–τ cycle.

7.3 Collapse without annihilation

Let V denote recursive volume (state count). Then

$$V(\mathcal{R}') = V(\mathcal{R}).$$

Collapse does not remove recursion; it reorganizes it. It compresses the recursion state manifold along specific directions, reducing complexity while maintaining state count.

7.4 Collapse triggers

Operator XII is invoked when:

- Φ and Ψ produce incompatible variational directions,
- the τ-field flow becomes tangent to multiple distinct surfaces,
- the variational domain becomes non-integrable,
- recursion tension exceeds a threshold determined by τ.

Mathematically, collapse is triggered when the kernel of ω_{UNNS} enlarges such that admissible variations are no longer independent.

7.5 Re-seeding recursion after collapse

After applying $\mathcal{O}_{\text{XII}}$:

- recursion is transferred to $\mathcal{R}'$,
- ω_{UNNS} is restricted to $\mathcal{R}'$,
- a new recursion potential θ'_{UNNS} satisfies

$$\omega'_{\text{UNNS}} = -d\theta'_{\text{UNNS}}.$$

- a new τ-field $\boldsymbol{\tau}'$ governs post-collapse evolution.

This provides a natural UNNS mechanism for *phase switching* between recursion regimes (e.g., quantum-like to geometric-like).

Chapter 8

Operator XII and Action Stationarity

8.1 Degeneration of the variational domain

Before collapse, the variational domain $\mathcal{V}$ consists of all smooth curves connecting fixed endpoints. After collapse, $\mathcal{V}$ shrinks to

$$\mathcal{V}' = \{\gamma' : \gamma' \text{ obeys constraints induced by } \mathcal{O}_{\mathrm{XII}}\}.$$

Variations that would move γ out of $\mathcal{R}'$ are forbidden.

8.2 Effect on action variation

Originally,

$$\delta\mathcal{A}_{\mathrm{UNNS}}[\gamma] = \int_\gamma \theta_{\mathrm{UNNS}} - \int_{\gamma'} \theta_{\mathrm{UNNS}}.$$

After collapse, this becomes

$$\delta'\mathcal{A}_{\mathrm{UNNS}}[\gamma] = \int_\gamma \theta'_{\mathrm{UNNS}} - \int_{\gamma'} \theta'_{\mathrm{UNNS}},$$

but only variations γ' respecting $\mathcal{O}_{\mathrm{XII}}$ are permitted.

The flux expression still holds:

$$\delta'\mathcal{A}_{\mathrm{UNNS}}[\gamma] = \int_\Sigma \omega'_{\mathrm{UNNS}}(\boldsymbol{\tau}', \cdot),$$

but Σ must lie entirely inside $\mathcal{R}'$.

8.3 Zero-flux under collapse

The UNNS stationary action condition becomes:

$$\delta'\mathcal{A}_{\mathrm{UNNS}}[\gamma] = 0 \quad \Longleftrightarrow \quad \int_\Sigma \omega'_{\mathrm{UNNS}}(\boldsymbol{\tau}', \cdot) = 0,$$

for all *collapse-compatible* Σ.

Thus, the physical trajectories after collapse are integral curves of $\boldsymbol{\tau}'$ instead of $\boldsymbol{\tau}$.

8.4 Interpretation of collapse

Collapse can be interpreted as:

- a constraint enforcement mechanism,
- a projection of recursion geometry,
- a reduction of variational freedom,
- a phase-reset in recursion dynamics.

8.5 Recovery of variational structure

After collapse, the action principle is regained, but on a simpler recursion manifold:

$$\mathcal{R} \xrightarrow{\mathcal{O}_{\mathrm{XII}}} \mathcal{R}',$$

with an updated symplectic-like structure:

$$\omega'_{\mathrm{UNNS}}, \quad \theta'_{\mathrm{UNNS}}, \quad \boldsymbol{\tau}'.$$

This concludes the formal treatment of Operator XII as a geometric variational reset mechanism.

Part V

Applications and Examples

Chapter 9

Low-Dimensional Recursion Examples

9.1 One-dimensional recursion example

Let x be a single recursion coordinate. Suppose recursion evolves by

$$\dot{x} = f(x).$$

In this trivial case:

- The recursion manifold is $\mathcal{R} = \mathbb{R}$.
- The counting form reduces to $\omega_{\mathrm{UNNS}} = \omega(x)\, dx \wedge dt$.
- The recursion potential may be written as $\theta_{\mathrm{UNNS}} = \theta(x)\, dx - H(x)\, dt$, where $\theta' = -\omega$.

The UNNS action becomes:

$$\mathcal{A}_{\mathrm{UNNS}}[\gamma] = \int (\theta(x)\dot{x} - H(x))dt.$$

Stationary action reproduces $\dot{x} = f(x)$.

This shows that *even in one dimension*, the UNNS variational structure collapses to the classical form when we project recursion onto a single degree of freedom.

9.2 Two-branch recursion example

Let recursive states be labelled by (x, y). Define the counting form as:

$$\omega_{\mathrm{UNNS}} = \omega(x, y)\, dx \wedge dy.$$

Define a recursion-flow field:

$$\boldsymbol{S}_\tau = (f(x, y),\ g(x, y)).$$

The physical trajectories satisfy:

$$\omega_{\mathrm{UNNS}}(\boldsymbol{S}_\tau, \cdot) = 0.$$

Explicitly:

$$\omega(x,y)\,(f\,dy - g\,dx) = 0.$$

This gives the recursion differential equation:

$$\frac{dy}{dx} = \frac{g(x,y)}{f(x,y)}.$$

Thus, even without any physical interpretation, the variational principle recovers the recursion-flow curves.

9.3 Interaction and bifurcation

If $f(x,y)$ or $g(x,y)$ change sign or vanish, recursion-flow bifurcations occur. These generate recursive analogues of:

- fixed points,
- attractors,
- separatrices,
- interference nodes.

All of these are resolved by Φ–Ψ–τ structure.

Chapter 10

Emergent Physical Theories

10.1 Quantum-like regime (dominant Ψ)

When $\|\boldsymbol{\tau}_\Psi\| \gg \|\boldsymbol{\tau}_\Phi\|$:

- recursion branches remain coherent,
- interference persists across recursion depth,
- ω_{UNNS} becomes strongly spectral,
- θ_{UNNS} resembles a phase-like one-form.

Effective projections into spacetime-like variables exhibit features of quantum mechanics:

- superposition,
- interference,
- phase evolution,
- decoherence only when τ increases.

10.2 Geometric regime (dominant Φ)

When $\|\boldsymbol{\tau}_\Phi\| \gg \|\boldsymbol{\tau}_\Psi\|$:

- recursion collapses into geometric sheets,
- ω_{UNNS} becomes curvature-like,
- θ_{UNNS} acts as a geometric connection form,
- trajectories resemble geodesics of an emergent metric.

The effective theory projects to classical geometry:

- gravitational curvature,
- classical causal structure,
- minimal interference.

10.3 Quantum–gravity crossover ($\tau \approx \tau_{\text{crit}}$)

At the critical τ scale:

- geometry and coherence compete,
- recursion is multidimensional,
- ω_{UNNS} encodes mixed curvature/coherence states,
- the variational principle requires the full UNNS form.

10.4 Role of Operator XII

Operator XII mediates transitions between recursion regimes:

- collapse of coherence $\to$ geometric phase,
- collapse of geometry $\to$ coherent phase,
- collapse of variational domain $\to$ new recursion sector.

This provides a natural UNNS mechanism for phase transitions that resemble:

- quantum measurement,
- classicalization,
- decoherence,
- geometrogenesis.

Part VI

Appendices

Appendix A

Appendix A: Construction of the UNNS Counting Form

We derive ω_{UNNS} from:

- independence of recursion directions,
- recursion conservation,
- compatibility with Φ–Ψ–τ cycles.

Choose recursion coordinates x^a. Define:

$$\omega_{\text{UNNS}} = \frac{1}{2}\omega_{ab}\, dx^a \wedge dx^b.$$

Closedness:

$$d\omega_{\text{UNNS}} = 0 \quad \Rightarrow \quad \partial_a \omega_{bc} + \partial_b \omega_{ca} + \partial_c \omega_{ab} = 0.$$

This ensures that recursion count is consistent under changes of surface.

Appendix B

Appendix B: Derivation of $\omega_{\text{UNNS}} = -d\theta_{\text{UNNS}}$

In a contractible region, closedness implies exactness:

- Poincaré lemma: every closed form is locally exact.
- Thus $\omega_{\text{UNNS}} = -d\theta_{\text{UNNS}}$ for some one-form θ_{UNNS}.

θ_{UNNS} is not unique:

$$\theta_{\text{UNNS}} \to \theta_{\text{UNNS}} + d\lambda$$

does not change ω_{UNNS}.

This freedom corresponds to choice of Lagrangian gauge.

Appendix C

Appendix C: The Condition $\iota_{\boldsymbol{S}_\tau}\omega_{\mathrm{UNNS}} = 0$

The interior product identity:

$$\iota_{\boldsymbol{S}_\tau}\omega_{\mathrm{UNNS}} = 0$$

means that $\boldsymbol{S}_\tau$ is always tangent to surfaces of constant recursion.

Equivalently:

$$\omega_{\mathrm{UNNS}\,ab}\boldsymbol{S}_\tau^b = 0.$$

This expresses that recursion does not flow across recursion-count surfaces.

Appendix D

Appendix D: Variation Surfaces and Flux Integrals

Let γ and γ' bound a surface Σ.

Flux:

$$\Phi_{\text{flux}}(\Sigma) = \int_\Sigma \omega_{\text{UNNS}}(\boldsymbol{S}_\tau, \cdot).$$

Use the identity:

$$\omega_{\text{UNNS}}(\boldsymbol{S}_\tau, \cdot) = -d\theta_{\text{UNNS}}(\boldsymbol{S}_\tau, \cdot)$$

and Stokes' Theorem:

$$\int_\Sigma d\theta_{\text{UNNS}} = \int_{\partial\Sigma} \theta_{\text{UNNS}}.$$

Appendix E

Appendix E: Comparison With Hamiltonian Mechanics

If recursion coordinates are split into (q^i, p_i, t):

$$\theta_{\text{UNNS}} = p_i \, dq^i - H_{\text{UNNS}} dt.$$

Then:

$$\omega_{\text{UNNS}} = dq^i \wedge dp_i - dH_{\text{UNNS}} \wedge dt.$$

If t-slices are considered, this reduces to the classical symplectic form and Hamilton's equations.

Thus, Hamiltonian mechanics is a *projection* of UNNS recursion.

Appendix F

Appendix F: Algebraic Properties of Operator XII

Operator XII satisfies:

- Idempotence on restricted sectors:

$$\mathcal{O}_{\mathrm{XII}}(\mathcal{O}_{\mathrm{XII}}(r)) = \mathcal{O}_{\mathrm{XII}}(r).$$

- Preservation of recursion count:

$$\int_{\mathcal{R}'} \omega'_{\mathrm{UNNS}} = \int_{\mathcal{R}} \omega_{\mathrm{UNNS}}.$$

- Compatibility with τ-flow:

$$\mathcal{O}_{\mathrm{XII}*}(\boldsymbol{S}_\tau) = \boldsymbol{S}'_\tau.$$

Appendix G

Appendix G: TikZ Code for the Φ–Ψ–τ Cycle

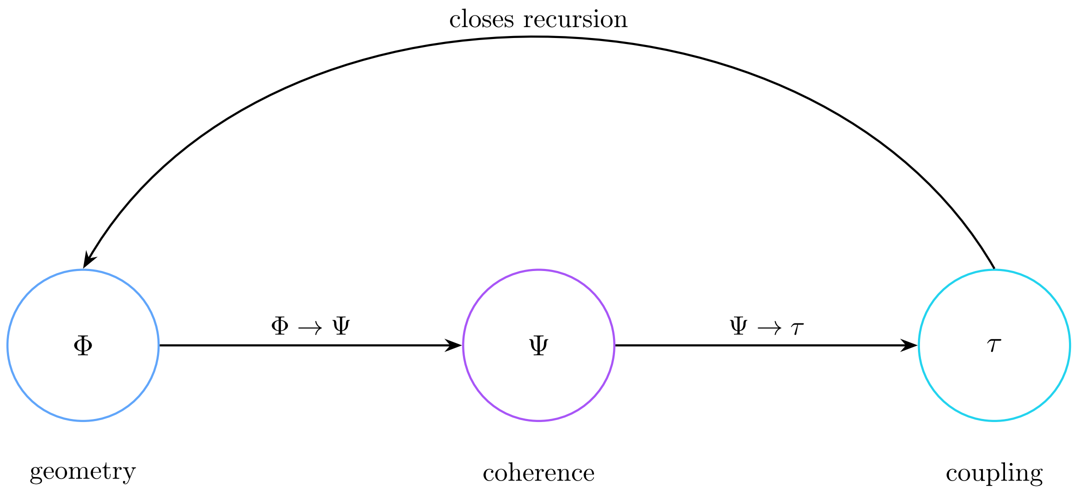

Figure G.1: Diagram used throughout the monograph.

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